Mathematics of Gambling / Edward O. Thorp

My family and I just returned from a twelve day cruise to Canada and Alaska. We traveled aboard the ship Pacific Princess, star of the TV series, *The Loveboat*. Daily, passengers were offered a variety of gambling situations similar to other cruise ships we've been on. I didn't see a sure way to "make a killing," but my ideas might suggest something to you. I think you will find some aspects of cruise ship gambling to be quite surprising.

Cruise Ship Gambling

Gambling included simulated horse races, bingo with a snowball feature, slot machines, pub night with blackjack tables, a ship's pool and private player-run poker games. The hustler or the expert player might profit from a high-stakes private game, such as poker, blackjack, bridge or backgammon. This was the only significant gambling opportunity that I noticed.

The Ship's Pool

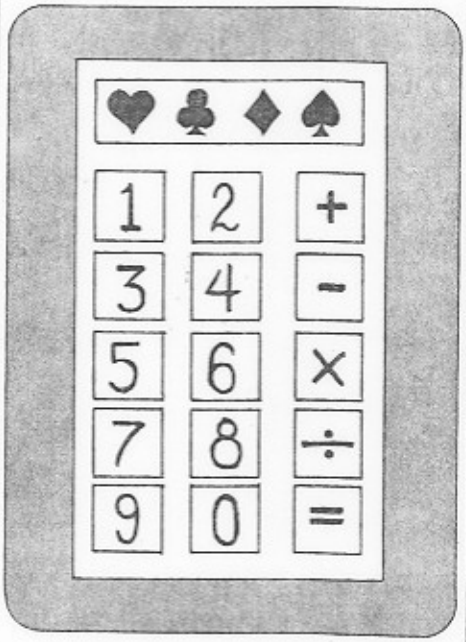
The ship's pool ran once a day. Players would bet on the ship's traveled distance between two specified times. On the day I checked it, there was a high field of 320 nautical miles or more and a low field of 300 nautical miles or less. You could bet on the high field, the low field, or on any individual number of nautical miles.

There were 19 bets: from 301 to 319 nautical miles, plus the two field bets. It cost one dollar per bet. Each bet was tallied on a large blackboard, indicating how many people were betting on each of the 21 choices.

They showed the navigator's estimate for the distance traveled, which was 316 nautical miles for that day. The winning number was 313 nautical miles. Unfortunately, the ship pool operators did not announce the percentage return of bettors' money. It is crucial to know this in determining whether or not to play.

Slot Machines

The slot machines ranged from $.10 to $1.00. Made by Bally, they were 3-reel machines. The players occasionally won, but generally appeared discouraged. One wealthy businessman, who pumped more than $2,000 into the dollar machines, was told by the operators that the slots average return was 84%. Without access to the insides of the machines, we had no easy way to verify this statement, but there is a method which might work.



Consider this idea: Suppose each of the machine's reels consisted of 20 alternatives. The operators can change the way the machine behaves by eliminating certain possibilities on each reel. Assume that the possibilities that have not been eliminated on each reel are equally likely.

You could make a machine act this way by punching a series of little holes in the side of each reel. Arrange them in a circle with one for each outcome (cherry, grape, lemon etc.). Then construct a little finger that fits into the holes. Hold the finger above the side of the reel when it is spinning. It can then be released at a random time allowing the finger to fall into the next hole.

Then, if you wanted to eliminate certain outcomes you could plug corresponding holes. When the little finger is released, it would fall into the next available unplugged hole. If the allowed outcomes on each reel are equally likely, and if you could determine the allotted outcomes, then you could ascertain the likelihood of various combinations. By looking at the payoff schedule for these combinations, you could estimate the return.

For example, suppose that each of the three reels has 20 outcomes. All the outcomes are possible and equally likely. Then there are 20 x 20 x 20 or 8,000 equally likely combinations. The only thing that pays off is the bells. There are two bells on the first reel, three bells on the second and five bells on the third. There are 2 x 3 x 5 or 30 ways that three bells can come up.

Now suppose that the three bells pay $200 on a $1 bet. If each of the 8,000 combinations came up once, 30 would pay $200 each or $6,000. The total bet amount would be $8,000. This is the expected long run result. Since 6,000 divided by 8,000 is 75%, the machine is returning 75% of the players money.

Now assume the house puts a plug in one of the 5 bells on the third reel. This gives you 20 x 20 x 19 or 7,600 equally likely outcomes. There are 2 x 3 x 4 or 24 ways to get three bells in 7,600 spins. On the average, the payoff is $4,800. The machine's return is now $4,800/$7,600 or about 63% of the players money.

Suppose you want to know how much a machine will return. If the

reels on the machine have enough dirt flecks, scratches, or other marks which will identify identical symbols from each other, then it is possible to build a "map" of each reel. From that map and the data that you collect to build it, you could probably tell the percentage returned. Since the machines are undoubtedly set against the player, building this map of the reels is just for fun and education. Don't expect to make money on it.

Here's how to proceed. After the reels have come to a stop, look into the little window and record the series of visable symbols for each reel. For example, on the first reel a bell might be right in the center of the window, just above it there might be a cherry, and just below it there might be a lemon. Look more carefully at these symbols and note any scratches, dirt marks, or other distinguishable characteristics. Do this for each of the three reels.

Then, insert a coin and spin the reels again. When they stop, jot down the same information for each reel. After several runs, you will see all the symbols on all the reels. You will notice groups of symbols. Take these groups and stitch them together to build up the circular pattern on each reel.

From the data collected, you can decipher which position arose. You can keep a tally similar to a series of three roulette wheels. After you have run the machine enough times, you can stipulate the frequency count on the "pockets" or symbols for each reel.

If any symbol was excluded, it will be apparent when enough data is collected. Once the excluded symbols are identified and deleted, you can use a chi-square test. Test the possible positions on a given reel and note if they are equally likely. (We have discussed this test in previous columns.)

If you know the possible numbers on each reel and if they seem equally likely, then use the payoff schedule given on the slot machine to determine the fraction of your return.

Many years ago, I carried out this mapping technique on two slot machines. I discovered that one of them was set at about 80%. The other one was a penny machine in front of a casino in downtown Las Vegas. It was set at about 105%. That machine was a cheap loss leader. It excited people and attracted them into the casino where the house made a profit on the other slots.

The 80% machine was a nickel machine that paid only 20 nickel jackpots. Once every 15 minutes for one minute the casino had "double jackpot time." During that one minute, it was a 160% machine. If you could play that machine for a good four minutes per hour, feeding in 100 nickles, you could make an average profit of $3.00 in an hour.

This was not worth anyone's time. It proved to be academic: Players filled up the machine for the other 14 minutes, and never left it open for the minute.

Don't play slots to win, play for fun.

In summary, don't play the slots to win, play for fun.

Blackjack

One night the ship's entertainment consisted of "English Pub Night." As part of the festivities, six blackjack tables were set up in the main entertainment area. Volunteer passengers served as dealers. My son and I were two of the volunteers.

The rules were dealer stands on all 17s, split any pair, double down on 9, 10 or 11 only. The maximum and minimum bets were $1.00. Jim Everett privately instructed that this prevented problems with those legendary beings known as card counters.

As dealer, I asked if anyone was a card counter or knew anything about card counting. No one seemed to.

We had our choice to deal a single deck or a double deck. The six tables operated for about 1½ hours in actual playing time and averaged about 4 to 5 players per table. We may have played 100 hands per player per hour. By multiplying 1½ hours x 4½ players x 100 hands x 6 tables, we estimated 4,050 hands that evening. That means $4,050 of initial bets. Let's round it off to $4,000. That might be $1,000 in error either way.

To avoid making change with half dollars, insurance was not allowed. The player had one very large benefit when the dealer did not have a blackjack: A player with blackjack was paid 2 for 1.

Beat the Dealer revised, page 131, shows that under these rules, the one deck basic strategy player's advantage is a little over 2%. The difference between one deck and two decks is just .35%, so the 2% figure applies as a rough approximation for either of the two games. From this, you would expect that the house would lose about 2% of $4,000 or $80 on an average if all players used good basic strategy. In fact, the house won $108 or about 2.7% of the total bets placed. This could be due to statistical fluctuations even though the players played well. The 20 players at my table and at my son's table played worse than the basic strategy player. They often split 10s or hit stiffs against small dealer up-cards, or stood on stiffs against big dealer up-cards.

Aside from the educational experience, how much money might you make in the long run per hour in such a game? Assuming there are 100 bets per hour at $1 each, a pure basic strategy player would average about 2% of that or $2.00 an hour. If you are a good counter and the deck is dealt nearly to the end, expect another $1 or more perhaps.

Horse Racing

The horse races were simulated by rolling dice. The track was a long roll of fabric divided into six strips. Each strip consisted of a series of rectangles or squares.

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Each horse marched along a line or series of squares. When the horses reached the end of the series, they would turn around and march back.

A typical race began with six horses lined side by side. A red die and a white die were rolled. The red die indicated which horse moved and the white die meant how far. If the red die showed three and the white die showed five, horse three would move five spaces.

The horses were wooden silhouettes attached to sticks on blocks of wood. One operator called out which horse moved and how far, and another operator moved them. Bets were one dollar each and you could make as many bets on as many horses as you wished. The operators took some money out of the pool, but never told us the amount.

On a previous cruise with a similar horse racing game, I asked about this. I was told that 20% was taken out for a marine charity fund. In this game, I determined that more than 20% was taken out, as we'll see.

During the course of a race the operators would count up the total amount of money collected. They would use this money to assign payoff odds to horses. Then, they would announce these odds during the course of the race. (To be continued next month.)

Q: *I refer to the April 1980 issue of* Gambling Times *wherein you program the HP 65 for the* Hi-Opt 1 *system.*

After entering your program, my display agrees exactly with your example. When I enter several more high cards (10 value), however, the display keeps increasing.

I'm not familiar with the Hi-Opt 1 *system, but it seems that with a succession of ten value cards, one's bet would decrease. The same holds true when the program is for 4 deck. Where am I going wrong?*

I.K.

P.S. I used the "Basic Strategy" suggested by you (and Revere) with surrender at Atlantic City as per J. Braun. With a progression of 1, 2, 5, 8, 12, 16, 20, 24 etc. (increasing only after a win),results have been as expected—about even—a large win offsetting many lesser losses. (Unit employed $25, never less, and occasionally $100.) Largest winning session, $18,000; largest losing, $3,000.

A: Congratulations on the fun you are having in Atlantic City.

You didn't go wrong with the HP 65 program. I did. When I checked my program against the column, I found that in writing the manuscript, following LBL C, I had sleepily repeated the part of the program following LBL B. Here is the correct version of the LBL C portion of the program with accompanying notes. Change "RCL 2 then z" to "RCL 3 then −H." Change "+ to" to "− to," and change "STO 2 z + 1" to "STO 3 −H−1."

Thank you for bringing this to my attention. gt