The House Take in the Options Markets

An investor (gambler) who trades securities on an exchange pays a commission according to a known schedule of rates. Studies of stock market systems, and of hypothetical performance, are more accurate when they include these costs. Some systems, which give the investor an edge before commission costs, lose their edge after such costs.

There is a second hidden transactions cost for investors which is seldom taken into account, but which may be of comparable magnitude. This arises from the market maker (or the specialist system, as the case may be), in use on the particular exchange considered.

When a member of the general public (i.e., anyone but a specialist or market maker) trades on an exchange, he may be trading with a market maker. This generally happens when there are no public orders near the last traded price. On average, the market makers make a profit on such trades. If they did not, they would not survive. Economists say this is a reward for the use of their capital and labor in providing a liquidity service to the market.

This gain to the market makers constitutes an equal loss to the public. It can be thought of as an effective additional commission cost borne on average by all the public participants.

I have just completed, along with Jerome Baesel and George Shows, research which estimates such costs to one institutional trader (a hedge fund) using data on actual options trades. The fund wanted to know what costs, if any, it was paying for the market maker function. The fund supplied data on actual options trades, involving 37,065 contracts with a value of $10,626,826. The total loss by the fund to market makers was estimated by us as $54,094, or 0.51%, with a 96% confidence interval being between −0.19% and +1.21%.

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We found the estimated loss to the market maker by the fund was more than three times as large on options traded only on the AMEX as it was for those traded only on the CBOE. The corresponding figures are 0.82% and 0.23%. The result is not quite enough to convince us the difference is significant, but it is suggestive.

We cannot discern whether the higher total cost on the AMEX was attributable to higher market maker "fees" or a greater fraction of public trades being with market makers. Readers who want details should write to the Graduate School of Management, University of California, Irvine, California 92717. Ask for Working Paper No. 2.

A Proposition Bet with Dice

I have a set of three dice. They are true dice in every aspect, but I have made one change. The first die, which I will call A, has two faces numbered 6 and four faces numbered 2. The second die, which I will call B, has four faces numbered 5 and two faces numbered 1. The third die, which I will call C, has three faces numbered 4 and three faces numbered 3.

I invite you to play a game in which we each pick one of the dice and roll it. We put one unit each in the pot. The higher roller wins the pot. The dice have been numbered in such a way that there will be no ties. Because I am very nice, I offer you the chance to pick the best die out of the three. I will pick one from the two left over.

My strategy is this: If you pick A, I will pick C. If you pick B, I will pick A. If you pick C, I will pick B. A little calculation shows the probability that A will beat B is 5/9. The probability that B beats C and the probability that C beats A are each 2/3.

Thus, we have the odd situation that even though you get to pick first and choose the "best" die, one of the remaining ones will always beat it. So, in fact, if I pick correctly I will always have the advantage.

I first saw this idea some years ago in a column by Martin Gardner in *Scientific American*. He called it the paradox of the non-transitive dice. The reason for the name is that in mathematics, a relationship among things is called transitive if it has the property: Whenever A beats B and B beats C, then A also beats C.

Now, with our three dice we see that A beats B more than half the time and B beats C more than half the time. But A beats C less than half the time. Most people find this contrary to their intuition.

For people that wish to explore this further, I have some questions for you to puzzle over. Send your answers to me in care of this magazine. I will print the best answers

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in future columns.

Question 1: In our example A beats B with probability 5/9, B beats C with probability 2/3, and C beats A with probability 2/3. These numbers add up to 17/9. Can you construct an example of three such dice in which the total of these three probabilities is still greater? If not, why not? If so, what is the largest total you can produce?

Question 2: Can you produce examples using four or more dice in which no matter which one is chosen first, there is another one which beats it more than half the time?

Question 3: Answer questions one and two when the dice have some different specified number of sides such as three, four and so on, with all faces having the same probability of coming up. To make such dice for any number of faces from three on up, proceed as follows.

Take a cylinder and shave sections parallel to its length so that you produce flat faces of equal size making equal angles to each other. You will end up with a cylindrical object having a regular polygon as cross-section. For example, to produce a four-sided die you end up with something that looks like a length of 2x2 lumber.

To produce a six-sided die you can take a hexagonal pencil and simply cut off the eraser end and the point end so that you have two flat ends perpendicular to the main faces. Then simply roll the pencil, having numbered the sides in any order you choose from one to six. Your impulse might be to count the uppermost face as the one which is scored.

However, since we allow an odd number of sides, two faces could be equally uppermost. As one face is always flat down on the table, a good rule is to always use the downmost face as the one that is scored.

Comment for mathematical readers: The dice A, B and C are physical representations of random variables. We can replace the dice in the game, and in the questions, by arbitrary random variables X, Y and Z. Suppose we have three "outcomes," each with probability 1/3.

For the first outcome define X=2, Y=1, Z=0. For the second outcome define Y=2, Z=1, X=0. For the third outcome define Z=2, X=1, Y=0. Then X beats Y, Y beats Z and Z beats X, each with probability 2/3. Thus, if Question 1 is applied to X, Y and Z, the answer is "at least 2."

The random variable idea suggests that we can use other chance devices as physical realizations of the random variables to construct more proposition bets along the same lines. For example, with cards we can imitate the dice proposition. Make three little packs of cards. Pack A has two 6s and 2s. Pack B has four 5s and two 1s. Pack C has three 4s and three 3s.

Instead of picking a die, you pick a pack. Then I pick a pack from the two you leave. We each shuffle our packs and draw a card. High card wins. The situation is identical to the dice example. **gt**