Mathematics of Gambling

Edward O. Thorp

The Horse Hedge Method

We will use last month's betting method for the cruise ship horse game to formulate some ideas for betting on real horse races. In this column, I will offer you some ideas which might help you develop your own system.

Unequal Probabilities

Last month's betting method provided situations in which we knew the true winning probabilities for each horse. Those probabilities are no longer necessarily equal in real racing.

Here is the more general rule that I call the horse hedge method: Suppose we have a pari-mutuel horse race with horses 1, 2, . . . , n and we know that the win probabilities are p(1), p(2), . . . , p (n) respectively. We'll bet unit p(1) on the first horse, p(2) on the second horse, etc. Below is the rule:

(1) For no track take, the expected profit per unit is $E = p(1) \times p(1) \times A(1)/A + \ldots + p(n) \times p(n) \times A/A(n) - 1$. It stays positive except when $p(1) = A(1)/A$, $p(2) = A(2)/A$, etc. The bettor will always have an advantage. The only exception is when the pari-mutuel pool is distributed among the horses according to the true odds. Using the hedge method in this case has no advantage or disadvantage; your money will simply be refunded.

(2) In case there is a track take, and a fraction K of the total pool A is returned, (making the total payback K x A), then the hedge bettor would have an advantage or a disadvantage given by formula $E = K\,[p(1) \times p(1) \times A/A(1) + \ldots + p(n) \times A/A(n)] - 1$. In this case, E is always greater than the track take, K-1, unless the pari-mutuel pool is distributed among the horses precisely in proportion to the true odds as in (1).

In that situation, the bettor does no better than the track take. When the pari-mutuel pool is allocated precisely according to the true odds, the hedge bettor will always do better than the average pari-mutuel bettor.

Will the same procedure work for unequal win probabilities as it did for equal chances in last month's column? It turns out that betting equal amounts on each horse in an unequal case may result in a greater or a lesser payback. It depends upon how much the allocated pari-mutuel pool deviates from the true odds. Below are examples that illustrate what can happen.

We can compute the expected gain per unit as 1/6, according to the horse hedge formula. If we bet equal amounts on each horse, the expected gain is 0.

Assume the track take is 1/14 of the total pool. That means that 13/14 of the total pool is returned and this is the value of K in Rule 2. We can use Rule 2 and the last column in Table 1 to calculate the expected gain for the horse hedge bettor. The result is 1/12.

When computing the expected gain for an equal wager amount, on each of the three horses, the loss is 1/14. This indicates that the horse hedge method improves the bettor's chances of overcoming the track take, whereas the equal betting method has an average loss equal to the track take.

Table 2 shows that the horse hedge method does better than the average bettor, as usual, but the equal bet method does worse than the average bettor. In Table 2 we assume no track take. The calculations show that the horse hedge method's advantage is 1/16 whereas the *equal bet method* has a disadvantage of 1/24. In this case, the methods differ more than 10%.

To calculate unequal win probabilities, number 11 horses from 2 to 12. Take bets on them and put them in a pari-mutuel pool with no track take. Determine which horse wins by rolling a pair of dice. The probabilites of the horses winning in 36th's are 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, respectively.

To apply the horse hedge method in this game, bet an amount pro-

TABLE 1

horse i	A [i]	p [i]	p [i] x p [i]	A/A [i]	K x A/A [i]
1	$1,000	1/2	1/4	3	39/14
2	$2,000	1/3	1/9	3	39/14
3	$3,000	1/6	1/36	3	39/14

TABLE 2

horse i	A [i]	p [i]	p [i] x p [i]	A/A [i]
1	$1,200	1/2	1/4	5/2
2	$1,000	1/3	1/9	3
3	$ 800	1/6	1/36	15/4

portional to these probabilities on each horse. For example, bet $1 on horse 2, $2 on horse 3, $3 on horse 4, etc. You will have an advantage if the pool is not allocated in the same order as your bets. In this game, your advantage depends on how much the pool allocation fluctuates from the true probabilities.

Track Take

With a track take, you do better than the average bettor. Whether or not you gain enough to have an advantage in the game depends on (1) the size of the track take (the smaller it is the better for you); and (2) the extent to which the pari-mutuel pool deviates from an allocation which is proportional to the true probabilities. The greater the deviations, the better for the horse hedge handicapper.

Why "Hedge?"

In the securities and finance markets, to hedge is to take two or more investment positions simultaneously. The risks should cancel out and an excess rate of expected return should remain.

In a real race, the true probabilities are not known. If we knew the true probabilities or had better estimates than the pari-mutuel pool offers, we might find horses with a positive expectation. Then we could simply bet directly on those horses instead of developing this method for the daily double.

There is a plausible argument which upholds the pari-mutuel pool's estimate of the true horse winning probabilities: "If there were a method for predicting horse winning probabilities, and these probabilities differed enough from the pari-mutuel pool's estimate to give the predictor an advantage, then he would place bets and by so doing would cause the pari-mutuel pool odds to shift in such a way as to reduce that advantage. With many bettors and much information and available computing power the overall effect is to reduce such advantages so they are small or even become disadvantages."

In other words, "If you could beat the casinos at blackjack, then they would change the game so you couldn't. Thus, there isn't any system for beating them."

If we assume that pari-mutuel pool probabilities are true probabilities then it is the one case where Rule 2 tells us the horse hedge system does not improve our edge over the track take! You might think that makes the horse hedge idea useless, but this is not true. Consider the daily double pool: The payoffs should be consistent with the probabilities in the individual race win pool; but in general, they aren't consistent. Thus, we have a chance to apply Rule 2 using probabilities based on the individual race win pool.

You can apply the horse hedge method to daily doubles, and with some modifications, you can apply it to exactas, and pick sixes.

The Daily Double

Let's apply the horse hedge idea to the daily double bet. The same idea, with some modifications, also applies to exactas, pick six and similar bets and to exactas, quinellas and trifectas in jai alai.

For a little background on the daily double, I quote from the book *Science in Betting: The Players and the Horses*, by E. R. Da Silva, and Roy M. Dorcus:

> *In daily double betting, any horse in the first race can be combined with any horse in the second race, and to win the bettor must successfully select the winners of both races. Some bettors combine all of the horses in the first two races. If there are ten horses in each race, in order to cover all possible combinations of horses, one would have to buy one hundred tickets at $2.00 each. If by chance long-odds horses won both races, it would be possible to make a profit on that single daily double. However, such a situation is not common throughout a week or a season. One daily double $2.00 ticket at Del Mar recently paid $2,878.60, another paid $685 and there were in addition three others during this season which paid over $200—yet actual returns for this season were only $6,808. The average number of horses was eleven in the first race and ten in the second. To have combined all these horses in all the daily doubles for this season would have cost $220 per race, and since there were forty-two days in this season, the total cost would have been $9,240, producing a loss of several thousand dollars.*

Notice that they consider betting equal amounts on each horse. From one season at Del Mar, they found that $9,240 in total bets were returned; $6,808 for a payback fraction of 0.74 or a loss of 26% of the amount bet. Thus, betting equal amounts on each combination did not work.

Illustrated below is the horse hedge method for daily doubles in a real race. Table 3 lists the winning probabilities based on odds for the first race at Del Mar on August 13, 1980. The horses are listed according to post position in the first column. The second column has the handicapping odds given in the *L.A. Times* on the morning of race day. The third column is obtained from these odds by taking the right hand number in the second column and dividing by the sum of the two numbers.

For example, 30-1 gives a probability of 1/31 = 0.0323. For the horse in the 13th post position, 7-2 gives a probability of 2/9 = 0.2222. When there is no track take, the probabilities calculated this way must add up to 1.00.

When there is a track take, the probabilities calculated from the final payoff odds at race time will equal more than 1.00. In fact, they add to 1/K, where K is the fraction of the pool, which is returned to the bettors. This rule is not quite exact due to the irregular effects of breakage, but the effects are generally small and not worth discussing.

In order to correct for probabilities that do not add up to 1.00, we add them, deducting horses which may have been scratched. We then use the final total and divide it into the preliminary probabilities so that it equals 1.00. (Corrected probabilities appear in column four.)

Column five gives the final odds on various horses. Column six has corresponding uncorrected probabilities and column seven lists corrected probabilities. Notice that column six adds to 1.2691; by dividing this into 1.00 we get 0.7880 which corresponds to a track take of 21.20% for this particular race.

Column three equaled 1.7237 before deducting the horses which were later scratched, making the track take too large. The sum for Del Mar is typically about 1.20; therefore the handicapper's setting of the odds was not consistent. On average, the odds were set too low in this race for the horses. When four horses were scratched, the odds on the remaining horses gave probabilities which equaled 1.3105.

TABLE 3

First Race

Horse [p.p.]	A.M. Odds [h.c.]	Prelim. Probs.	Corrected Probs.	Final Odds: 1	Prelim. Probs.	Corrected Probs.
1	30-1	0.0323	.0246	114.80	.0086	.0068
2*	4-1	0.2000	.1526	3.10	.2439	.1922
3	2-1	0.3333	.2544	0.70	.5882	.4635
4	20-1	0.0476	s	s	s	s
5	5-1	0.1667	.1272	7.30	.1205	.0949
6	6-1	0.1429	.1090	12.10	.0763	.0601
7	10-1	0.0909	.0694	24.00	.0400	.0315
8	30-1	0.0323	.0246	72.60	.0136	.0107
9	30-1	0.0323	s	s	s	s
10	8-1	0.1111	.0848	6.40	.1351	.1065
11	15-1	0.0625	.0477	52.50	.0187	.0147
12	20-1	0.0476	.0363	82.50	.0120	.0094
13	7-2	0.2222	s	s	s	
14	8-1	0.1111	s	s	s	
15	10-1	0.0909	.0694	81.20	.0122	.0096
	sum	1.7237				
After scratches		1.3105	1.0000	1.2691	1.0000	
		Estimated track take			21.20%	

TABLE 4

Second Race

Horse [p.p.]	A.M. odds [h.c.]	Prelim. Probs.	Corrected Probs.	Final Odds: 1	Prelim. Probs.	Corrected Probs.
1*	7-2	.2222	.1725	2.60	.2778	.2313
2	15-1	.0625	.0485	18.80	.0505	.0421
3	3-1	.2500	.1941	5.90	.1449	.1207
4	6-1	.1429	.1109	13.30	.0699	.0582
5	5-2	.2857	.2218	3.80	.2083	.1735
6	6-1	.1429	.1109	15.80	.0595	.0496
7	10-1	.0909	.0706	2.80	.2632	.2192
8	10-1	.0909	.0706	6.90	.1266	.1054
	SUM	1.2880	1.0000		1.2007	1.0000
		Estimated track take			16.72%	

That is the typical sum at Del Mar.

Table four presents the probability calculations for the second race. The fourth column appears to equal 0.9999, but shows 1.0000, because the entries have been rounded off to four places.

The final outcome of the daily double: horse 2 won the first race; horse 1 won the second race; and a winning $2 ticket paid back $38.60 or $19.30 per unit bet. The amount bet on each of the 15 x 8 or 120 combinations is proportional to the product of the corresponding probabilities.

For example, if we use the corrected probabilities based on the morning odds, we have .1526 for horse 2 in the first race and .1725 for horse 1 in the second race. The product of these two numbers is .0263. That means we bet .0263 of our total unit bet on the combination which actually won the daily double. Therefore, we have a return of $19.30 x this probability or .5080 of a unit which means we lost 49.2% of our bet. If we had used the final odds, the probabilities are .1922 and .2313. Their product is .0445 and we would receive this amount x $19.30 or .8580 of a unit, or a loss of 14.2%.

On page 127, Da Silva and Dorcus warn you that:

> *In doing any statistical work on daily doubles, the reader must be careful not to use the actual closing odds of the horses, as listed the day following the races in result charts from newspapers or from the Form or the Telegraph, since these last-minute odds are not available to the daily double bettor for either the first or the second races. The bettor must rely only upon the probable odds for statistical study of daily double betting, odds which are given in the Morning Line at the tracks, in the Form or Telegraph under different handicappers such as Sweep, Analyst, Trackman, or given in the track programs.*
>
> *Furthermore, in dealing*

with these probable odds, the bettor must remember that they may or may not correspond to the last-minute closing odds on the toteboard.

(For the first race only, the actual odds that we would use in practice may be fairly close to these final odds if we were actually at the track watching the toteboard.) At this point, we can see difficulties with the horse hedge idea as it relates to the daily double.

Keep a record of final odds to overcome problems.

For example, there is a minimum $2 bet. In order to approximate the various probabilities of the typical one hundred or so combinations, we have to make several hundred $2 bets which requires a substantial bank roll. Another problem is that the final pari-mutuel pool odds are unknown. Even if we did know the odds on the individual races, the true probabilities of the individual horses winning in their respective races would still be unknown. Therefore, we don't know if the horse hedge method will give us an advantage over the track take.

TABLE 5

Uncorrected Probability Sums

Race #	8/13/80	8/14/80	8/15/80
1	1.2691?	1.1970	1.2177
2	1.2007	1.1968	1.2104
3	1.2064	1.1973	1.2026
4	1.1960	1.2049	1.2059
5	1.2082	0.9966?	1.1986

Even if it does give us an advantage, we don't know if we can gain enough to overcome the track take for an overall advantage. This is the reason why this system needs further development.

One way to get around the difficulties is to keep a record of the final odds and the corresponding probabilities and bet accordingly. If pari-mutuel odds are a fair estimate of true odds, then this indicates the sort of gain to be had from horse hedging. If the gain is large enough to produce a substantial advantage, then there might still be an advantage if we use good odds that are available to us at the time we place our bets.

To show you how to keep this sort of record, I will use one average figure to correct for the track take. Table 5 shows the sum of the uncorrected probabilities for the first five races on three consecutive days. The days are August 13, 14 and 15, 1980 at Del Mar. The two entries followed by question marks suggest that there may be data errors or newspaper misprints. Except for the two questionable figures, the uncorrected probability sums are close to 1.20. The average, of the 13 remaining races in Table 5, works out to be 1.2033.

To simplify, I shall use 1.20 in my computations in Table 6. The fractions estimate the investment returned for each day the horse hedge system is used at Del Mar. If you want to construct a similar table, get extensive racing records from your track, and determine whether the method works over a past sample.

Table 6 shows the idea at Del Mar. The second, third, and fourth columns list the corrected probabilities based on the final odds of 2-1 for the winning horse in the first race. The fifth, sixth, and seventh columns do the same thing for the second race. The eighth column is the product of these prob-

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TABLE 6

Del Mar

Race Date	Race 1 Winner Odds: 1	Prelim. Prob.	Corrected Prob.	Race 2 Winner Odds: 1	Prelim. Prob.	Corrected Prob.	Product of Probs.	Daily Double Payback for 1	Fraction Returned
8/13/80	3.10	.2439	.2033	2.60	.2778	.2315	.0470	19.30	.9080
8/14/80	2.50	.2857	.2381	.50	.6667	.5556	.1323	6.50	.8600
8/15/80	0.70	.5882	.4902	1.90	.3448	.2874	.1409	5.50	.7748
8/16/80	1.80	.3571	.2976	2.30	.3030	.2525	.0752	12.90	.9695
8/17/80	5.50	.1538	.1282	6.70	.1299	.1082	.0139	50.30	.6979
8/18/80	1.60	.3846	.3205	9.50	.0952	.0794	.0254	41.60	1.0582
8/20/80	0.90	.5263	.4386	5.30	.1587	.1323	.0580	17.80	1.0327
8/21/80	2.20	.3125	.2604	6.40	.1351	.1126	.0293	45.50	1.3343
8/22/80	1.70	.3704	.3086	11.60	.0794	.0661	.0204	36.90	.7532
8/23/80	7.40	.1190	.0992	1.10	.4762	.3968	.0394	27.60	1.0865

Ten races: estimated average fraction returned 0.9476

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abilities (the pari-mutuel estimate of the probability of a pair of horses winning the daily double). For the last column, multiply the payback on $1 which is the fraction of the unit bet returned to us.

In our sample of ten races, we get an estimated payback of 94.76%, or a loss of 5.24%. We are estimating the average effective track take as 1-1/1.2 = 16.67% so the system does better than average but still does not win.

For a clear explanation of daily double betting, exactor or exacta betting, odds, and trifecta betting, I refer you to the appendix of *Harness Racing Gold,* by Prof. Igor Kusyshyn, published by International Gaming Inc., 1979 ($14.95).

According to the book *Beating the Races With a Computer,* by Steven L. Brecher, the New York Racing Association take out is currently 14% although it has been 17%. Brecher also says the California take out is 15.75%. Of course the effect of breakage is to increase the average take out somewhat beyond these figures.

Readers who want to know more about the calculation of winning probabilities based on the pari-mutuel odds should read Chapter 3 in *Horse Sense,* by Burton P. Fabricand, published by David McKay and Co., 1965. The book is hard to obtain, but I believe you can find it in the larger libraries.

Fabricand takes a sample of 10,000 races, with 93,011 horses and 10,035 winners (some dead heats). He finds that the average loss, from betting on the favorites (high pari-mutuel probability of winning), is considerably smaller than the average loss from betting the long shots (low pari-mutuel probability of winning).

For extreme favorites, the sample showed a profit and for horses with a pari-mutuel win probability of 30% or more the average loss was just a few per cent. It ranged gradually higher as the odds lengthened for horses with odds of 20 to 1 or more and pari-mutuel probabilities averaging about .025; the average loss to the bettors was 54 per cent.

This indicates that the odds, from the pari-mutuel pool for winners, are systematically biased; they can be improved by incorporating a correction factor based on a data sample similar to Fabricand's. The correction would increase the probabilities assigned to the favorites and decrease the probabilities assigned to the long shots systematically.

A more readily available source for the same information is Fabricand's latest book *The Science of Winning,* published by Van Nostrand Reinhold in 1979. On page 37, a table shows how a player's expectation varies with the odds. The sample has 10,000 races with 10,035 winners because of dead heats. We'll conclude the hedge idea next month. **gt**