The Cost of Liquidity Services in Listed Options: A Note

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Several researchers claim to have detected inefficiencies in the market for listed put and call options [9, 15, 16, 18, 27]. However, these studies have ignored the implicit costs of dealing with a market-maker. Phillips and Smith [20] estimated these additional costs from quoted bid-asked spreads and concluded that when market-maker costs were added, otherwise profitable trading rules no longer yielded excess returns. On the other hand, a related study by Beebower and Priest [4] finds no such market-maker costs for the NYSE.

To resolve the above, data from a private investment partnership (hedge fund) covering 1,894 actual options transactions were analyzed. These trades took place over a one-year period and involved 37,065 contracts with a value of over $10 million. Examination of these trades suggest that the percentage cost to the fund for market-maker services is .54% of the value of the options traded.

I. Structure of Options Markets

The theory of market-making divides market participants into two classes: market-makers and members of the public. In Figure 1, $S(p)$ and $D(p)$ represent traditional supply and demand curves. Their intersection, the equilibrium price $P_e$, represents the price a willing seller could expect from a willing buyer if the assets were placed on the market and the seller was willing to wait a reasonable period of time.

$S(m)$ and $D(m)$ represent “always present” supply and demand curves of the market-maker. $P_b$ represents the price at which a member of the public could sell if he took the market-maker’s bid and $P_a$ represents the price at which a member of the public could buy immediately from the market-maker if he were willing to pay the market-maker’s asking price.

Both AMEX and CBOE option markets have market-makers. The AMEX option market has a specialist system (see [1]) in which one member of the

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1 Our data are available to researchers wishing to extend or verify this analysis.

2 The theory of market-making stems from the work of Demsetz [12] and has had a substantial development [3, 5, 12, 19, 20, 22, 23, 24]. Tinic and West [25] present the central ideas.

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exchange acts as the center of the market for all options on a single stock. The specialist, like all other exchange members, may participate in any trade and one of his functions as a market-maker is to ensure that there are prices at which others can either buy or sell. The amount of business the specialist does will depend upon whether the prices he quotes are competitive relative to other quotations. These may be from other exchange members or may be public orders. The specialist keeps this information in his "book" so that he generally has information about the market that his competitors lack.

The CBOE has a competitive market-making system (see [8]). Any member of the exchange who is in the crowd at the post where a particular option is traded can make a market in the option. No one holds a "book" which provides him with market information unavailable to others.

II. The Model

Consider a market which includes both the public and market-makers. Trades may be classified into three types: a purchase by a member of the public from a market-maker; a trade between two members of the public; or, a sale from a member of the public to a market maker.

Since the market-maker must make a profit, the prices at which trades between market-maker and the public take place should differ from the equilibrium prices in a direction adverse to the public. To represent this, we write the price at which the public buys, $P_B(t)$, as

$$P_B(t) = P_e(t) + Z_B(t)$$

(1B)
where \( P_e(t) \) is the equilibrium price and \( Z_{S}(t) \) is a random variable. If the public buys from the market-maker the trade, on average, includes a premium so the expected value of \( Z_{S}(t) \) is greater than zero.

Similarly, we write any trade price \( P_S(t) \) at which the public sells as

\[
P_S(t) = P_e(t) + Z_{S}(t)
\]  

(18)

Then \( Z_{S}(t) \) is the discount from the equilibrium price which the public takes to get the market-maker to buy. On average, \( Z_{S}(t) \) will be negative.

Since market-makers must eventually sell as much as they buy, it seems plausible to assume that a random trade is as likely to be at a premium as at a discount. If we further assume that, on average, the market-maker premiums on sales to the public equal discounts on purchases from the public,\(^3\) then it follows that \( P_e(t) \) is on average equal to the observed prices. Therefore, we assume that on a randomly selected trade the observed price is an unbiased estimate of the equilibrium price at that time. In particular, this is true for the transaction day’s closing price, \( P_e(t) \).

Therefore, to estimate the implicit cost of trading with the market-maker, we used the closing price on the day of a trade as an estimate of the equilibrium price at the time of the trade, i.e.,

\[
\hat{P}_e(t) = P_e(t)
\]  

(2)

Aside from a small bias due to drift,\(^4\) the equilibrium price at the close should be a reasonable estimate of the equilibrium price at the time of the trade. The estimated percentage cost of a trade is then taken as \( 100(P_{B}(t) - P_e(t))/P_e(t) \) for a buy and \( 100(P_e(t) - P_{S}(t))/P_e(t) \) for a sale.

Average liquidity cost is not an estimate of the discount asked by the market-maker, but an estimate of the expected liquidity cost. This is the product of the liquidity cost and the probability of trading with the market-maker. Since that probability is unknown, it is not possible to estimate the discount or premium the market-maker gets when he participates. It is possible only to estimate the overall cost for this fund in its dealings over the year in question.

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\(^3\)This assumption may hold on average, but there is no reason to expect it to hold in specific cases. Archibald, Baesel, and Brewer [2] model optimal premium and discount quote strategies. They show that the optimal bid-ask spread quote for a market-maker may tilt or shift around the equilibrium price so the premium is not equal to the discount. The tilt is a function the market-maker’s inventory. Without knowledge of dealer inventory we consider the “no tilt” assumption plausible and useful for our current analysis.

\(^4\)We assume that stock prices will tend to rise over time in order to compensate the owner for foregoing consumption. The relationship between the equilibrium price \( P_e(c) \) of an asset at the close, \( c \), of a day and some time, \( t \), during the day will, in general, depend on the time until the close. If the upward trend is linear, then at time \( t \), \( E(P_e(c) = P_e(t)(1 + m(c - t))) \), where \( m(t_1 - t_2) \) is the expected drift between time \( t_1 \) and \( t_2 \), in this case between the time of the trade and the close. Conversely, we can observe \( P_e(c) \) and \( E(P_e(t)) = P_e(c)/(1 + m(c - t)) \). Since total returns to stockholders have historically averaged 8–12% annually, the expected intraday drift in equilibrium prices should be less than \( 12%/365 \) or 0.033%. Suppose that on average about half this drift occurs between the time of the trade and the close. Then the average value of \( m(c - t) \) should be less than 0.017%. Both these figures are small compared to the implicit costs we find, so we tend to ignore the drift.
III. Execution Procedure

The options traded were part of a diversified portfolio of options hedged against a diversified stock portfolio. The most important factor in option selection was mispricing as determined by a variant of the Black-Scholes’ [6] option valuation model; a second consideration was the degree to which the option would help the fund in its diversification.

Each morning traders receive a list of options evaluated with the aid of the Black-Scholes option model. Orders are then entered for options with apparent mispricing greater than a specified cut-off level. Orders are typically contingent limit orders good for the day. These orders specify a limit price on the option contingent on the underlying stock’s price being at a particular price or better.

IV. The Results

Table I shows the results from 1,894 option transactions involving 37,065 contracts traded between October, 1978 and October 1979. The average loss to the market maker is reported in Column (2) of the table. For all options, the mean loss was .54%. This was significantly different from zero (using a standard, one-tailed t-test) at the .05 level. We conclude that, on average, the fund did pay money to the market-maker.

The table also classifies average cost by put or call and buy or sell transactions.

<table>
<thead>
<tr>
<th></th>
<th>(1) Number of Transactions</th>
<th>(2) Estimated Average Cost</th>
<th>(3) t-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Options</td>
<td>1894</td>
<td>.54%</td>
<td>1.63</td>
</tr>
<tr>
<td>CBOE &amp; AMEX</td>
<td>190</td>
<td>-.96%</td>
<td>-1.16</td>
</tr>
<tr>
<td>CBOE-Not AMEXb</td>
<td>1197</td>
<td>.64%</td>
<td>1.42</td>
</tr>
<tr>
<td>AMEX-Not CBOEb</td>
<td>507</td>
<td>.86%</td>
<td>1.59</td>
</tr>
<tr>
<td>Buy Puts</td>
<td>112</td>
<td>4.41%</td>
<td>2.73</td>
</tr>
<tr>
<td>Sell Puts</td>
<td>55</td>
<td>-.06%</td>
<td>-.02</td>
</tr>
<tr>
<td>Buy Calls</td>
<td>761</td>
<td>2.38%</td>
<td>4.18</td>
</tr>
<tr>
<td>Sell Calls</td>
<td>966</td>
<td>-1.32%</td>
<td>-3.31</td>
</tr>
</tbody>
</table>

* The number of transactions n is substantially understated. The reason is that the fund’s option orders are typically executed as several distinct transactions during the day. That happens because the acceptable option prices depend on the current stock price and because the fund, in order to trade in the volume it desires, is often forced to do so in a series of trades. At the end of the day, separate trades in the same option at the same price are aggregated and recorded as a single “transaction.” We estimate that the true n values in Column (1) of the table are at least twice as large as those we report. Consequently, the t-statistics in Column (3) are at least $\sqrt{2} = 1.41$ times as large as those we give. This makes the significance level higher in Column (3). In particular, the cost would appear to be significantly non-zero not only for “buying puts,” “buying calls,” and “selling calls,” but also for “all options,” the “CBOE-Not AMEX” and “AMEX-Not CBOE” listings.

b Transactions involving options trading on one exchange but not on the other.
Buying puts and buying calls had positive average costs at 4.41% and 2.38%, respectively. Both are significant at the 99% confidence level with t-statistics of 2.73 and 4.18. Selling puts and selling calls had negative average costs at −0.06% and −1.32%, respectively. The selling puts category contained only 55 observations and the result is not significantly different from zero. The average cost for selling calls has a t-statistic of −3.31 and is significantly negative at the 99% confidence level. A negative market-maker cost implies the investor is “paid” for participating in the trade. While the negative cost may be a chance event or result from measurement error, the possibility should be considered that the fund was being encouraged to write options.

In general, dealing in puts was more expensive than dealing in calls. Buying puts, in particular, was the most expensive category in the table. Since the time of these trades, however, the put markets have matured and have much greater volume. The difference between put and call liquidity cost might not exist today.

Our estimate of the fund’s cost of market-makers’ services for an average trade is 0.54% of the value of the trade. The mean bid-ask spread for the Phillips and Smith data (their Table II, Page 185) for calls selling over 50c was 4.51%. The corresponding put spread was 5.77%. For comparison purposes, these figures must be divided by two as our numbers focus on the mark-up or mark-down on one side of a round-trip. From the Phillips and Smith estimates this is about 2.5%. Their results can be reconciled with ours by recalling from our earlier discussion that the expected cost considers not only the market-maker’s “fee” (which is what Phillips and Smith estimated) but the probability of dealing with the market-maker.

Phillips and Smith estimate that the market-makers participate in about 20% of trades and earn about 2.5% of the value of the trade. From this we would expect our procedure to yield an estimate of .5%, which is approximately what we observe. This reasoning may also help explain the Beebower and Priest [4] finding of (approximately) zero market-maker cost for stocks on the New York Stock Exchange. Phillips and Smith estimate the average bid-ask spread on NYSE stocks to be .62% (Table II, page 185). The “one-way” cost would be about .3%. New York Stock Exchange specialists participate in 11–12% of the trades [19, p. 12]. If the specialist participated in 12% of the Beebower and Priest trades and earned .3% on the trades participated in, we would expect to observe an average liquidity cost of .036%. This value is smaller than those mentioned in previous discussions of liquidity cost because it is an average or expected cost rather than the cost of actually using a market-maker. Further, the standard deviation of costs (measured as the difference between transactions price and equilibrium price as estimated by the closing price) will be large because of price variability between the time of the trade and the close of the day. A large standard

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5 This is not the first time negative liquidity costs have been reported. Beebower and Priest [4, page 39] estimated liquidity costs of .15% for selling stocks and .12% for buying stocks. They concluded this result was not highly significant. Archibald, Baezel, and Brewer [2] developed optimal bid-ask quote strategies for market-makers concerned with risk and return when stock price changes contain both diffusion and jump components. In their model, controlling inventory risk caused market-makers to quote spreads which lead to negative costs for some investors trading with them.
deviation makes it difficult to identify such a small cost as "significant" without a very large sample.

We conclude that the evidence is consistent with the existence of market-maker costs of trading options, but that the expected cost may be much smaller than indicated by bid-ask quotes.

Phillips and Smith made the important observation that the market-maker cost must be considered in studies of option market efficiency. They estimated the market-maker spread, but did not take account of the fact that only a fraction of trades incur the full spread penalty. This fraction may depend upon the trading strategy used. Our results suggest that the difference between the spread and the expected spread may be substantial for certain trading strategies and that the strategies "discredited" by Phillips and Smith should be re-examined using a more realistic estimate of liquidity costs.

REFERENCES