

# The Mathematics of Gambling

## Systems for Roulette III

by Edward O. Thorp

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The general principles we discussed last month apply to almost all gambling games, and when they apply, they guarantee that systems cannot give the player an advantage.

To help you reject systems, here are conditions which guarantee that a system is worthless:

### A Set of Conditions Which Make Systems Worthless

I. Each individual bet in the game has negative expectation. (This makes any series of bets have negative expectation.)

II. There is a maximum limit to the size of any possible game. (This rules out systems like the no-limit doubling up system discussed in the January/February issue.)

III. The results of any one play of the game do not "influence" the results of any other play of the game. (Thus, in roulette, we assume that the chances are equally likely for all of the numbers on each and every future spin, regardless of the results of past spins.)

IV. There is a minimum allowed size for any bet. (This is necessary for the technical steps in the mathematical proof. Most people would take for granted that there is such a minimum, namely some multiple of the smallest monetary unit. In the U.S.A., the minimum allowed bet is some multiple of one cent. In West Germany, it may be some multiple of the pfennig, and so forth.)

Under these conditions, it is a mathematical fact that every possible gambling system is worthless in the following ways:

- (1) Any series of bets has negative expectation.
- (2) This expectation is the (negative) sum of the expecta-

tions of the individual bets.

- (3) If the player continues to bet, his total loss divided by his total action will tend to get closer and closer to his expected loss divided by his total action.
- (4) If the player continues to bet it is almost certain that he will:
  - (a) be a loser;
  - (b) stay a loser forever, and so never again break even;
  - (c) eventually lose his entire bankroll, no matter how large it was.

To give you an idea of how valuable this result is for spotting

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We have devised a powerful test for proving that a system can't win

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worthless systems, here are some examples of systems which cannot possibly give the player an advantage:

1. All the roulette systems I have ever heard of, except the following two types. (a) Biased wheels, in which condition (I) may be violated; the numbers are no longer equally likely, so bets on some numbers may have positive expectation. (b) Physical prediction methods, in which the position and velocity of ball and rotor are used to predict the outcome. This is discussed elsewhere in this month's issue by Charley De Lisle on page 38, and will be discussed starting next month in this column.

2. All craps systems I have ever heard of, except possibly those using either crooked dice or physical "control" of dice.

3. Any systems for playing keno, slots, chuck-a-luck, the wheel of

fortune, and the money wheel.

As a further illustration, consider the book *Gambling Systems That WIN*, published by Gambling Times, 1978, paperback, \$2. Of the fourteen systems given there, our result applies at once to eight. (The other six are one blackjack system, four racing systems, and a basketball system.)

(In the case of sports bets, it is generally difficult to determine whether condition (I) is satisfied. In the case of blackjack, condition (I) fails if the player counts cards, and there are, in fact, some winning systems, as most of you know.)

This leaves eight systems in *WIN*: four craps systems, one baccarat system, two roulette systems, and a keno system.

Conditions (I) through (IV) hold for all eight systems so none of them are winning systems. Nor do any of them reduce the house edge in the slightest. However, they may be entertaining. Also, in games like keno, craps, and roulette, where the expectation may vary from one game to another or from one type of bet to another, some ways to bet are "smarter" (translation—less dumb; more accurate translation—less negative expectation but still losing) than others.

For those who are prepared to lose, but want to lose more slowly, such systems may be of interest.

In most cases, the basic information is a list of the various bets in the game and their expectation. Then, if you must play, choose only bets with the least negative expectation. The "system" complexities and hieroglyphics are not essential.

It may amuse you to see why

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## JUNIOR MEAD

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down by a couple of good players and learned a lesson in discipline. Jake was frightened but didn't show it.

"No dealer's choice," said Junior. "Just straight five-card stud. A gambler's game, right, Mr. Hilliard?"

Jake wanted to figure out his style of play, so he folded the first couple of hands just to watch. Junior bluffed Tom on a four card flush in one hand, and Jake noticed that he smiled at Tom while he waited for him to call. Jake would remember that and use it if he could. On another hand he noticed when Junior had three sixes, he rubbed his index finger against his thumb waiting impatiently and hoping that Mike would call with two pair.

Whenever Junior had a very good hand, he would take a fist full of money and rush his opponent into betting.

After about three rounds, Jake figured he had enough mental notes on Junior to play. He had the deck in his hand, but he wasn't quite sure how he should play. Should he hustle Junior? Bluff him? Heavy-bet him with money? How much did Junior know? Jake asked himself again and again.

Jake decided to try a stack to see if Junior could catch it. Four players... aces with two cards between them... add two... count off four... add two more... put a crimp in and put the deck down for Mike to cut. Mike caught the crimp, and Jake watched Junior's eyes. He couldn't tell whether or not Junior knew.

At one-fifteen the following afternoon, Jake Hilliard, tired, unshaven and yawning, got up from the table. Junior Mead sat smugly fingering the money he had won off the three men. "Well, Mr. Hilliard, what do you have to say now?"

"You fooled me, Junior," said Jake. "I thought you couldn't play, but you can. You're not the best, but you're damn good. I figured I could get you when it got down to just you and me, but that's when you really showed me something."

"It's like they always tell the defeated man, Mr. Hilliard," said

Junior, "you're good too, but you're only second-best as long as I'm around."

"I haven't heard that in a long time," said Jake. "A long, long time."

There was a heavy nip in the air, and Ellie closed the car window and rested back as Jake drove home. She took Jake's hand and examined it. She looked at him and laughed aloud. She placed his hand back on the steering wheel and asked, "Why did you lose, Jake?"

Jake thought a moment. "Junior Mead is a good poker player."

After a sigh and a pause, Ellie asked again, "Why did you lose, Jake?"

"I told you," he answered, without turning to her.

"Okay then, who did you call last night?"

"Call?"

"Yes, during the break we went home and you asked me to fix some coffee, and you went upstairs and made a phone call."

"Oh, yeah. I forgot. I wanted Steve to pick up some lotion."

"Jake, darling!"

"Yes."

"Since when do you call Steve at two-thirty in the morning? You know the phone bill will come in and I'll see eventually where you called."

Jake was silent. Ellie moved closer to him. "We're all packed," she said. "That's why I left the game early, to go home. I had it figured out the minute you started to lose. I haven't lived with you for ten years for nothing, you know."

Jake smiled.

"How's Cat Willis?" Ellie asked.

Jake stopped the car. "Oh, honey, Cat is going to tear Junior apart. I'll admit it took me longer than usual to figure him out, but once I did, it was like taking candy from a baby. I had to lose. It might have been better cutting him down myself, but Junior isn't quite ripe yet. But when he gets to New York with all those bright lights and fancy girls, oh, he'll be ready then. And Cat will rip him to shreds. I'd love to see it. Do you really want to go, Ellie?"

"Hell, man," said Ellie, "I'd much rather see that than see a three-legged mouse tap-dancing in sneakers." 9

## ROULETTE SYSTEMS

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condition (IV) is needed. Suppose, instead, that there is no minimum bet and that we are playing Red at roulette. Our first bet is \$1,000. There is an 18/38 chance that we win \$1,000 and a 20/38 chance we lose \$1,000. Now suppose that the second bet is \$0.90, the third bet is \$0.09, the fourth bet is \$0.009, the fifth bet is \$0.0009, etc. (Remember: no minimum.) Then the total of all bets from the second on is  $0.99999\dots = \$1.00$ . The total gain or loss on these bets is between  $-\$1.00$  and  $+\$1.00$ . The total action on all bets is  $\$1,000 + \$1 = \$1,001$ .

If we won the first bet, our total winnings (T) will always be between \$999 and \$1,001. This happens with probability 18/38. Therefore, conclusions 4(a), 4(b), and 4(c) fail. Also, our total action is \$1,001 so T/A is always between \$999/\$1,001 and \$1,001/\$1,001. But our expected gain (E) is negative so E/A is less than 0. Therefore, if we win the first bet, T/A does not tend to get closer and closer to E/A. Therefore, conclusion (3) also fails.

Conclusion 4(c) also deserves some comment. Actually, there is an insignificantly small chance the player can win the casino's bankroll before losing his. But even for moderate-size casino bankrolls, this possibility is so tiny as to be negligible, no matter how large the player's bankroll! We will discuss this in detail in a later column on "The Problem of Gambler's Ruin." It is also discussed at some length in the 1962 edition of my book, *Beat the Dealer*, and in Feller's great "An Introduction to Probability and its Applications, Vol. I," Wiley. Thus, a more exact version of conditions I-IV would include information about the size of the casino bankroll. Then conclusion 4 would include information about the tiny chance that 4(a), (b), and (c) don't happen.

As far as I know, the most elementary mathematical proof ever given for all this is in my textbook, *Elementary Probability*, available from Robert E. Krieger Publishing Co., Inc., 645 New York Avenue, Huntington, New York



11743. The proof is outlined on pp. 84-85, exercises 5.12 and 5.13. It requires no calculus and can be followed by a good high school mathematics student if he works through pp. 1-85.

We now have a powerful test for showing that a system doesn't win. This keeps us from wasting our money and time buying or playing losing systems. It also helps us in our search for systems that do win, by greatly narrowing the possibilities.

Now we turn to a discussion of how it is possible to win at roulette by predicting the outcome with physics.

In *Beat the Dealer*, Revised, page 181, I wrote in 1966, "There are also several people (including myself) who possess a method for beating roulette wheels whether or not they are defective!

"I played roulette on a regulation wheel in the basement lab of a world-famous scientist. We used the method and steadily averaged 44 per cent profit. In an hour's run, betting no more than \$25 per number, we won a fictional \$8,000! There are certain electronic problems which have so far kept the method from being used on a large scale in the casinos. (The few times I have used it to turn two or three dimes suddenly into a pile of silver dollars has caused enormous excitement.)

"The method works, and the story behind its discovery and development is a long and fascinating one. It will be even more fascinating when, sometime in the next few years, some of the few who possess the idea cash in on it in the casinos." Over the years I have had hundreds of letters and calls about this from readers.

I generally indicated what it was I did and encouraged the others to proceed. Several groups of people subsequently developed the apparatus and techniques to beat roulette. Coincidentally, the article "We Beat the Wheel!" in this magazine, by Charley De Lisle, tells the story of one of these groups. As I told the editors when they queried me, "As a result of the roulette comment about my prediction method which appeared in *Beat the Dealer*, Revised, I have talked on the telephone

and in person to several people who have worked to construct a similar system. I specifically remember talking to people in Santa Barbara, which is consistent with the author's comments. The details which the author gave of his group's implementation seems authentic to me and leads me to believe that this is a true account. . . . The group seems to have carried out the method well.

"The phrase . . . 'Thorp vaguely discusses a roulette wheel that a friend of his had wired up in his basement' is not accurate. We set up a regulation wheel in a basement laboratory and studied it with MIT lab equipment. We did not 'wire up a wheel.' The basic methodology we used, including tilts, was published by me in brief form in my paper "Optimal Gambling Systems For Favorable Games" (which appeared in the Review of the International Statistical Institute, 1969).

" . . . The author speculates that camouflage was the problem which kept us from winning. Not so! The two problems were difficulty with read-out devices and also (as in the case of the author's associate), my associate and I were doing so much better in other activities, like the market, that we lost most of our motivation." (See page 11 of Rev. ISI.)

And then, as I was finishing this column, *Gambling Times* forwarded this letter to me. I have encoded the author's name so the casinos would not identify him.

Dear Professor Thorp:

I am in hopes that G.T. will forward this to you soon; since I don't know your address, I must depend on them.

I have just read your "Systems for Roulette 1" article in the new G.T. (Jan.-Feb '79). My heart sank as I saw your subject for the third article, "3. Prediction using physics." I can only believe that you would reveal this to the gambling public if you have reason to believe that it is already out of the bag, and that no one will be hurt by it. Please believe me, that is not true.

I cannot implore you strongly enough, please, please, don't reveal the principles or details of your predictions as to the section of the wheel where the ball will finally

Continued on next page

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stop. Surely you don't need the money from the article, or the prestige, for that matter, since you have already gained the world-wide respect you have for your work in gambling.

The casinos aren't taking any great counter-measures against roulette players at this time, and I am afraid that if you give the principles away, it will create enough of a stir that it will no longer be a source of income for those who have devised the "physics method." I fully expect you to get letters from many of us begging you not to end it all for us—a lot of money is involved, I'm sure, and I can't see why you would even care to reveal any information, unless you're working for the casinos. They are doing quite well under the present circumstances, and the few of us who have the methodology aren't really hurting anyone—if you give it away, you will hurt some people. Please, give thorough thought to it and make your article vague enough to leave the physics alone. It is so simple that almost anything you write will lead to the proper conclusions, I'm afraid, but if you don't write something, now that you've published the first article, there will be a furor.

Thank you,  
AACELON

Readers can write me directly at the Mathematics Department, University of California, Irvine, CA 92664. There are too many individual letters for me to answer but I will respond to a very few, and will respond to some in this column.

Dear AACELON,

I can understand your concern at my coming extended revelation of the physics method. But as you see from this issue, others are writing about it. And I've already disclosed it in 1966 (BTD), 1969 (Rev. ISI), and so has Richard Epstein in his revised edition. I also have already made a prior commitment to my readers to deliver them this information. For me this takes priority.

Perhaps you can be consoled with this thought: I announced the discovery of winning blackjack systems in January 1961, with—as it happened—enormous publicity. Yet even now, 18 years later, it is possible (but admittedly much more difficult) to win.

Next month I will begin discussing the physical prediction of roulette outcomes. 98

## PUTTING IN THE FIX

Continued from page 74

together and had their cohorts bet against the 21-point spread. The second team, however, was not informed of the conspiracy, and they laid their money on Sing Sing to win by more than 21 points.

In the game that followed, Sing Sing led by 20 points late in the second quarter. Then strange things began to happen—the first string began making all kinds of boo-boos. The subs started hollering to get into the game, but they were waved off the field. Finally, mercifully, the half ended.

At this crucial point coach Law returned and found his first and second teams arguing away. He was still unsure of the true cause, so he sent the first team in for the second half. After the first fumble, however, he put in the second team who eventually put the game on ice with a final 50-0 score.

O'Reilly concluded that "the repercussions that followed caused trouble for months and finally football (at Sing Sing) was queered forever. It was one of the most outrageous betting coups in gridiron history."

The blackest moment in American sports history, though, occurred in 1919. That was the year baseball's Chicago White Sox threw the World Series to the Cincinnati Reds. Eight players were brought to trial in 1921, and even though the jury acquitted them (of attempting to defraud the public), baseball commissioner Judge Kenesaw Mountain Landis announced: "Regardless of the verdict of juries, no player who throws a ball game will ever play professional baseball!"

And eight talented young men never played in the big leagues again. The motive behind their actions was, of course, money, and it was well known in baseball circles that the Sox owner, Charles Comiskey, was a tight-fisted man. (The money for the fix was put up by the "Big Bankroll"—New York gambler Arnold Rothstein.)

It should be noted that this was a more innocent America—the scandal rocked the nation. These men had been entrusted with an almost sacred trust. Sure—some