The Mathematics of Gambling

Systems for Roulette I

by Edward O. Thorp

In this month's article, as well as in the two following, I will discuss three main ideas for roulette systems:

1. *Mathematical systems.* There is no "mathematical" winning system for roulette and it is impossible ever to discover one.

2. *Beating biased wheels.* Actual roulette wheels may become biased or defective in a variety of ways, possibly enough so that some numbers favor the player.

3. *Prediction using physics.* Claude Shannon and I built a transistorized electronic analog computer which successfully forecast the section of the wheel where the ball would finally stop. This gave a winning edge in laboratory and casino tests.

To clearly understand and appreciate what systems might win in roulette, it is important first to understand why no "mathematical" system can ever be devised. When mathematicians analyze the game of roulette, they assume that each of the numbers has the same chance beforehand of coming up on any one spin of the ball and wheel. To fix the discussion, let's consider the standard American wheel. This has thirty-eight numbers, namely, 0, 00, 1, 2, . . . 36.

The mathematician's *assumption*, that each of these numbers is equally likely beforehand to come up on any spin of the ball and wheel, seems plausible. The wheels are carefully machined and balanced by the manufacturer. They are checked from time to time by the casinos. When they show signs of wear they may be thoroughly reconditioned. Even if the wheel has irregularities which make some numbers more favored than others, if the player does not know this and his system is not designed to exploit this, then mathematical reasoning—based on the assumption that all numbers are equally likely to come up—gives correct conclusions about that player's system.

By a "mathematical system" I mean a system where all numbers *are* equally likely to come up on each spin and the player decides what bet to make using only the following information: (1) a record of what numbers have come up on some number of past spins, and (2) a record of the bets he has made, if any, on those spins. We must also assume there is a smallest allowable house (minimum) bet (such as $1) and a greatest allowable house (maximum) bet (such as $1000).

Casinos need to fix a maximum bet in order to stop the simple mathematical system of "doubling up." To see why, imagine we've found a casino with no maximum. We bet $1000 on Red. There are 18 Red numbers so our chance to win is 18/38. The payoff if we do win is $1000, because Red pays even money or 1 for 1. If we lose, we double and bet $2000 on the second turn. If that wins, we net $1000 on the two turns. If the second bet loses, we double again and bet $4000 on the third turn. Having lost $3000 on the first two turns, a win of $4000 on the third turn nets $1000 on the cycle of three turns. We continue doubling our bet after each loss. Finally, when we win, we have a net gain of $1000. We put this $1000 safely aside and start a new cycle of doubling until we win with a bet of $1000 on the Red. Each completed cycle wins another $1000 net. Table 1 illustrates this cycle.

The doubling-up system in Table 1 with no casino limit on bets is being discussed *not* because anyone would ever be allowed to do it, but to illustrate ideas we will be using. To see how ridiculous the system would be, note that if the first ten turns of a cycle have lost, on the eleventh turn the player bets 1,024 times his initial bet. His initial bet was $1000, so he bets $1,024,000. Of course the chance is small that this will happen. The last column shows a chance of 0.9984 that the cycle ends on or before the tenth turn, hence that the eleventh bet is never made. Thus, the chance of reaching the eleventh turn is only $1-0.9984=0.0016$ or 0.16% or about one chance in 613. But if the doubling-up system is used long enough, it will happen.

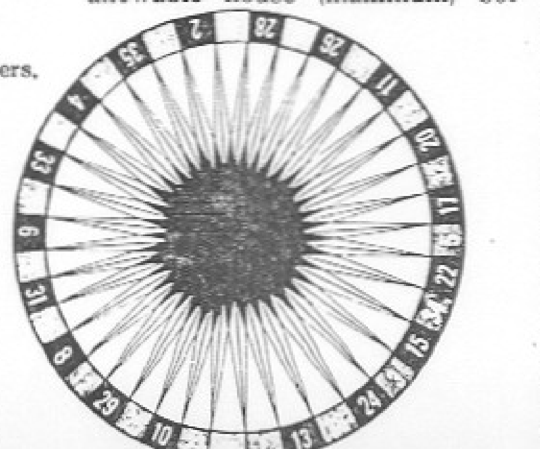

Figure 1. Arrangement of numbers, standard American wheel

Table 1. One cycle of the doubling-up system when the casino has no maximum bet.

turn #	amount bet	total profit if cycle ends on this turn	chance cycle ends on or before this turn: exact	decimal approximation
1	\$ 1,000	\$1,000	$1-(20/38)$	0.4737
2	\$ 2,000	\$1,000	$1-(20/38)^2$	0.7230
3	\$ 4,000	"	$1-(20/38)^3$	0.8542
4	\$ 8,000	"	$1-(20/38)^4$	0.9233
5	\$ 16,000	"	$1-(20/38)^5$	0.9596
6	\$ 32,000	"	$1-(20/38)^6$	0.9787
7	\$ 64,000	"	$1-(20/38)^7$	0.9888
8	\$ 128,000	"	$1-(20/38)^8$	0.9941
9	\$ 256,000	"	$1-(20/38)^9$	0.9969
10	\$ 512,000	"	$1-(20/38)^{10}$	0.9984
11	\$1,024,000	"	$1-(20/38)^{11}$	0.9991
31	$\$1000 \times 2^{30}$ or about a trillion	"	$1-(20/38)^{31}$	0.999,999,997,7
36	$\$1000 \times 2^{35}$ or about 34 trillion	"	$1-(20/38)^{36}$	0.999,999,999,9
100	about $\$6 \times 10^{32}$	"	$1-(20/38)^{100}$	
n	$\$1000 \times 2^{n-1}$	\$1,000	$1-(20/38)^n$	

With 30 losses in a row, the player is supposed to bet about one trillion dollars on the thirty-first turn. This exceeds the net worth of the New York Stock Exchange. On turn 36, the bet is about \$34 trillion. This exceeds the net worth of the world! (The net worth of the U.S.A. is about 6 trillion current dollars. I'd guess the net worth of the world to be about \$30 trillion.) The player should arrange from the start to have unlimited credit, *reasonably* pointing out that since he must eventually win he is sure to pay off!

Real casinos don't go for this. They have house limits (which they may increase sometimes under special circumstances) and credit limits. So this "sure-fire winning system" is never used. But players for centuries have used modified doubling-up systems in actual casino play. An illustration is given in Table 2. Here the player starts by betting \$1 on Red. He keeps doubling his bet until he wins. Then he starts the cycle over with a \$1 bet on Red. Each cycle produces a \$1 profit *unless*—and here is the catch—he loses 10 times in a row and then wants to bet \$1024 on the eleventh turn of the cycle. The house limit prevents that and prevents further doubling if the player loses on his eleventh turn.

Notice from Table 2 that if the player wins after 9 or fewer losses, he wins \$1 and successfully completes the cycle. But if he loses 10 times in a row, he can bet only \$1000 on the eleventh turn. If he then wins, he loses "only" \$23 on this cycle. But if he loses on the eleventh turn, he loses \$2023 on the cycle, for a major disaster. Of course, the chance of ever reaching the eleventh turn of a cycle is, as we saw before, only about one chance in 613.

Question: Is this sytem any good, or do the chances of loss on the eleventh turn ruin it?

Answer: We are going to find out that the "house percentage advantage" on Red is not changed *in the slightest* by the doubling-up system. In fact, the disaster of the eleventh turn is *exact* compensation to the casino for the high chance the player has of winning \$1 per cycle. We will show this by a computation. But what is perhaps truly amazing is that this is also true for all mathematical systems, no matter how complex, including all those that can ever be discovered. Since there are an infinite number of such systems, we cannot prove this by computations (an infinite amount of time would be needed to do the required infinite number of computations). In-

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Table 2. The doubling-up system when the maximum bet is \$1000 and the minimum bet is \$1.

turn #	amount bet	total \$ losses before bet	net profit if cycle ends, this turn	chance of this result: exact	decimal approximation
1	1	0	1	$18/38$	0.4737
2	2	1	1	$20/38 \times 18/38$	0.2493
3	4	3	1	$(20/38)^2 \times 18/38$	0.1312
4	8	7	1	$(20/38)^3 \times 18/38$	0.0691
5	16	15	1	$(20/38)^4 \times 18/38$	0.0363
6	32	31	1	$(20/38)^5 \times 18/38$	0.0191
7	64	63	1	$(20/38)^6 \times 18/38$	0.0101
8	128	127	1	$(20/38)^7 \times 18/38$	0.0053
9	256	255	1	$(20/38)^8 \times 18/38$	0.002789
10	512	511	1	$(20/38)^9 \times 18/38$	0.001468
11	1000	1023	-23	$(20/38)^{10} \times 18/38$	0.000773
			or -2023	$(20/38)^{11}$	0.000858
				total = 1	

ROULETTE SYSTEMS

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stead, I will indicate how the mathematician, by logic (like the logic of, say, plane geometry with its axioms, theorems and proofs) can show that none of this infinite number of systems is any good.

A lot of what I'm saying is easier than it sounds. For instance, to see that there are an infinite number of systems for roulette, all I have to do is give you any endless list of systems. Here is one such list (always bet on Red): System 1. Bet $1 on Red if Red came up one turn ago; if it didn't come up one turn ago, bet $2. System 2. Always bet $1 on Red if it came up two turns ago; if it did not come up two turns ago, bet $2. And so on for systems 3, 4, . . . etc. I didn't say my list of systems would be interesting, only that it would be endless!

The doubling-up system can be good for some fun even if it doesn't alter the house edge. Men, suppose you're in Las Vegas with your wife or your date. It's almost dinner time and you say casually, "Dinner for two will run us about thirty dollars. Why don't we eat for free? I'll just pick up $30 at this roulette wheel. It'll only take a few minutes." If you have $2100 in your pocket and the house limits are from $1 to $1000 on Red, you can use the doubling-up system. You need to complete 30 cycles without ever having a string of eleven losses. You will win $1 per cycle, for a total of $30, and be off to dinner.

How safe is this scheme? What are your chances? Table 1 says that the chance a cycle lasts 10 turns or less, and therefore you win $1, is 0.9984. The chance that you do this 30 times in a row turns out to be 0.9984^{30} or 0.9522, so the chance you succeed is over 95%. If you set your sights lower, say $20 or $10, then the chances of success go up to 96.79% and 98.38%, respectively. But be warned: if you fail, you can lose as much as $2023.

An important factor in determining the risk of failure is the ratio of the house maximum bet on Red to the minimum bet. To illustrate, suppose instead of $1 to $1000 for a ratio of 1000, the betting limits were $2 to $500, for a ratio of 500/2=250. Then if we start a cycle with a $2 bet, we hit the house limit on the ninth spin, after eight losses. (To see this, use Table 2 and double all the numbers in the second, third and fourth columns, because we *start* with a $2 bet rather than a $1 bet, as before.) Now the chance the cycle ends in eight turns or less is (from the last column of Table 1) 0.9941. Thus to win $30 you need to complete 15 cycles, the chance of which is 0.9941^{15} or 0.9152. If you try this in a roulette game with better odds, say single-zero American style or, still better, single-zero European style, the chance of success increases.

The doubling-up system is one of a class of systems that are sometimes called martingales. The origin of the term is given in the American Heritage Dictionary, New College Edition, which is the most informative definition I have seen on this. The word evolved from a similarly named village of Martigues in the Provence district of southern France, whose residents were viewed as peculiar and were roundly ridiculed with Gallic expertise. Their bizarre behavior included such things as gambling with the doubling-up system and lacing up their pants from behind. To use the doubling-up system became known as gambling "a la martigalo" (fem), "in the Martigues manner," i.e., "in a ridiculous manner."

There are many other popular "mathematical" systems. "Tripling up," where the player bets 1,3,9,27, etc. until he wins, then repeats, is like doubling up, but it wins faster and runs into trouble (in the form of the house limit) faster.

If you want to know more about "mathematical systems," consider these books:

The just-published book *Casino Gambling, Why You Win, Why You Lose,* by Russell T. Barnhart (Brandywine, N.Y., 1978, $12.95). Barnhart is a skilled magician and a longtime student of gambling. He has gambled extensively all over the world so he knows both the theory and practice of his subject. The book has 50,000 spins from an actual wheel and an elaborate discussion of mathematical or "staking" systems.

Allan Wilson's classic *Casino Gambler's Guide* has considerable material on systems and their fallacies. His treatment of biased roulette wheels may be the best ever written; we shall be referring to it later.

Richard Epstein's engaging treatise, *The Theory of Gambling and Statistical Logic, Revised,* (Academic Press, 1977) is a landmark in the subject. Much of it requires a university-level mathematics background. However, it is the best single reference work in print on the general subject of games and gambling, and even the general reader can glean much from browsing through it.

Next month I'll explain why mathematical systems, like the doubling-up system, cannot reduce the casino percentage. **9**ß

ADVANCED CONCEPTS

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that clichéd advice about not playing with scared money didn't apply to me because I didn't let my emotions affect my game.

Parking in front of my apartment, I stayed in the car for several minutes. The worst part, I thought, is that I can't quit. I must keep playing. And I must keep increasing the stakes and finding more games. But where am I going to find money to live on and play poker, much less to pay the checks I wrote tonight? I'm broke. I've sold all my personal property, including my gun . . . can't even shoot myself.

Next Month
Advanced Concepts
versus
Common Concepts

Milton unsuccessfully tries to raise money for his poker games. But even without money, he organizes a big game for Monday night at his apartment. He then gains strength and confidence by studying the next Advanced Concept. **9**ß

GOOD ADVICE

Don't lose your head!
It may be all you have left
when the game is over.

Laurie Dawson
Nashville, TN