Solution of a Poker Variant

EDWARD O. THORP

REPRINTED FROM INFORMATION SCIENCES

VOLUME 2 · NUMBER 3 · JULY 1970

Solution of a Poker Variant

EDWARD O. THORP
Mathematics Department, University of California at Irvine
Irvine, California

Communicated by Frank B. Cannonito

ABSTRACT

In one variation on poker, each player is dealt one card which he places exposed on his forehead. Thus each player knows every other hand but does not see his own. We call this variation "inverse poker". We show that, under certain restrictions, two-person inverse poker and two-person ordinary poker are isomorphic games. In particular, the existing solutions to two-person poker variants all yield solutions to two-person inverse poker variants.

1. THE ISOMORPHISM

We begin with the isomorphism between the ordinary poker variant [1, 19.4–19.6; 2, pp. 13–30] and a form of inverse poker. Notation is as in [1], to which it will be necessary to refer in order to follow the exposition. The ordinary and inverse games both begin with the two players each posting an ante b. Then they each receive a hand, a number $s = 1, \ldots, S$, each having the same probability $1/S$. In the ordinary version, we denote the hands of players 1 and 2 by s_1 and s_2, respectively. In the inverse version, we denote the hands by t_2, t_1, respectively. In the ordinary game player 1 sees s_1, which he also holds, and player 2 sees s_2, which he also holds. In the inverse game, player 1 sees t_1, which is held by player 2, and player 2 sees t_2, which is held by player 1. The subsequent betting and payoffs are as in the ordinary game.

Now let $t = \phi(s) = S - s + 1$, which is a one-one correspondence of $1, 2, \ldots, S$ onto itself. Then ϕ induces a one-one correspondence between $\Sigma_k(i_1, \ldots, i_S)$, the set of pure strategies in the ordinary game for player k, and $\Sigma_k'(i_1, \ldots, i_S)$, the set of pure strategies in the inverse game. The correspondence is

$$\Sigma_k(i_1, \ldots, i_S) \to \Sigma_k'(i_{\phi(1)}, \ldots, i_{\phi(S)}).$$

Information Sciences 2 (1970), 299–301

Similarly, ϕ induces a correspondence between the matrix elements of the ordinary and the inverse games given by

$$H(i_1, \ldots, i_S | j_1, \ldots, j_S) \to H'(i_{\phi(1)}, \ldots, i_{\phi(S)} | j_{\phi(1)}, \ldots, j_{\phi(S)}).$$

Now observe that

$$\begin{aligned} H'(i_{\phi(1)}, \ldots, i_{\phi(S)} | j_{\phi(1)}, \ldots, j_{\phi(S)}) &= \frac{1}{S^2} \sum_{t_1, t_2 = 1}^{S} \mathscr{L}_{\mathrm{sgn}(t_2 - t_1)}(i_{t_1}, i_{t_2}) \\ &= \frac{1}{S^2} \sum_{t_1, t_2 = 1}^{S} \mathscr{L}_{\mathrm{sgn}(s_1 - s_2)}(i_{t_1}, i_{t_2}) \\ &= \frac{1}{S^2} \sum_{s_1, s_2 = 1}^{S} \mathscr{L}_{\mathrm{sgn}(s_1 - s_2)}(i_{s_1}, i_{s_2}) \\ &= H(i_1, \ldots, i_S | j_1, \ldots, j_S). \end{aligned}$$

Thus corresponding matrix elements are equal in the two games so they are isomorphic as games. In particular, to translate an optimal solution of the ordinary game into one for the inverse game, a player of the inverse game takes the hand h which his opponent holds and plays as though he himself held $\phi(h)$ in the original game.

One sees intuitively and can verify by similar procedure that the ordinary (continuous) game of [1, 19.7–19.10.4] is isomorphic to an inverse game via $\phi(z) = w = 1 - z$. The discussion of solved variants of ordinary two-person poker given in [1, 19.11–19.16.2], [2, pp. 30–50], [3] and numerous other sources carries over via $\phi(s) = t = S - s + 1$ for the cases of finitely many distinct equiprobable hands and via $\phi(z) = w = 1 - z$ for the cases of hand distributed uniformly on [0, 1].

If the hands are not equiprobable or uniformly distributed, the isomorphism carries over if we remember to induce a transformation of the probability distribution of hands, as well.

2. LIMITATIONS OF THE METHOD

If the form of ordinary poker under consideration has a "draw" phase, we encounter difficulties in forming an isomorphic inverse poker. How shall a player draw when he does not know what he holds? This can be resolved in principal by having each player announce his drawing procedure in all circumstances. A referee then makes the appropriate draw and gives the player his new hand. This can be done in a way which gives the player no information, even whether or not he has drawn. However, it may be impractical to fully specify the player's drawing procedure.

This establishes the

Duality principle for two-person poker: For each version of two-person poker there is an isomorphic version of two-person inverse poker (and conversely).

The difficulty in extending the isomorphism to more than two players arises from the fact that each player sees a subset of hands, rather than a single hand, and that the various subsets seen on a deal are probabilistically dependent, unlike the hands in the two-person case.

REFERENCES

1 J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior* (Science Editions), Wiley, New York, 1964.

2 D. Gillies, J. Mayberry, and J. von Neumann, Two variants of poker, *Annals of Mathematics Studies*, Vol. 28, Princeton University Press, Princeton, N. J., 1953.

3 S. Karlin, *Mathematical Methods and Theory in Games, Programming and Economics*, Vol. I. Addison-Wesley, Reading, Mass., 1959.

Received February 6, 1970