May '81 70

The Mathematics of Gambling

by Edward O. Thorp

Roulette: The Dealer's Signature Fallacy

Q: *I received in the mail an advertisement for a roulette system from a "Dr. No Zero" in Las Vegas. He claims to have a Ph.D., but writes his blurb in simplespeak. Be that as it may, he refers to an article in the December* **Gambling Times** *by Stephen Kimmel, which describes a roulette system based on the predictable behavior of the dealer.*

Kimmel argues that a dealer who works eight hours a day, 50 weeks a year tends to spin the ball in the same way each time and usually imparts about the same velocity to the rotor each time he spins it. Kimmel presents a graph in which is recorded the number of spaces on the wheel that lie between the ball's resting places after each spin. His graph shows a peak at about 28 pockets; Kimmel suggests that bets placed between 21 and 30 pockets beyond where the ball last landed would have a 50 percent chance of success.

Dr. No Zero claims to have greatly improved the Kimmel system and offers to sell his method for a moderate sum. What do you think of Kimmel's system? Do you believe that Dr. No Zero has, in fact, got something even better?

N.Y.C.

A: I don't believe Kimmel's approach works, and I think that if you buy Dr. No Zero's system, you will zero out. Here's why: there are three important conditions that must remain roughly constant throughout play for the player to take advantage of the regularity of the dealer or, as Kimmel calls it, the dealer's signature. These conditions are (1) the rotor velocity should be approximately the same each time the ball is spun, (2) the spinning ball should make approximately the same number of revolutions each time, and (3) the initial position of the rotor when the dealer launches the ball should be approximately the same each time. This third condition, which is not mentioned in Kimmel's article, is crucial.

By way of illustration, suppose that the rotor velocity was exactly the same each time and that the dealer spun the ball exactly the same number of revolutions in each instance. Suppose further that the ball spun exactly eight revolutions and the rotor four revolutions during this time. Given those assumptions, the ball would land about 12 revolutions beyond the point where it was launched. In other words, if the number 13 was passing the ball as the dealer released it, the ball would arrive 12 revolutions later, *relative to the spinning rotor*, at approximately the number 13. You can see, however, that if the number 2 on the rotor was closest to the ball at the instant it was released, the ball would then end up near that number 12 revolutions later.

If the dealer releases the ball without regard to which number on the spinning rotor is closest to the launch point, the ball would randomly fall on the rotor 12 revolutions later. In this case, there would be no predictability whatsoever, even though the rotor velocity is absolutely fixed and the number of ball revolutions constant. Any variance in rotor velocity or number of ball revolutions would further guarantee a random outcome. Because Kimmel did not discuss variations in the point of release, I do not believe in his method.

There is a better approach to this statistical analysis of roulette. Watch a dealer and count the number of revolutions the ball makes on the stator from the time of release until it crosses onto the rotor. Note how constant that number of revolutions is. The results of your observations can be statistically stated as some average number of revolutions plus an error term.

Next, count the number of revolutions the rotor makes during the time the ball is on the stator. This will give you another average for the number of rotor revolutions, plus a second error term. Finally, count how far the ball travels on the rotor after it has crossed the divider between the rotor and stator. You can summarize these results as some average number of revolutions or pockets plus an error term.

In order for this approach to work, it is necessary that the square root of the sums of the squares of the error terms be less than 17 pockets. The proof of this is in my October 1979 column on roulette, in which a table shows what the rate of return is, given various root mean square errors. That table demonstrates that a positive return is possible only when that root mean square error is less than 17 pockets. (Note to mathematicians: I am using the normal approximation for the statistical discussion. I think it is very nearly an accurate description of what happens and that this approximation only slightly affects the discussion.)

Now for the improved method. In the unlikely event that the root mean square error is less than 17 pockets, then—and only then—you have a chance to win. The key lies in using the position of the rotor when the ball is launched as your starting point for predicting where the ball will fall out on the wheel.

For example, suppose you find that for a certain dealer the ball travels eight revolutions with a root mean square error of five pockets. Suppose also that during this time, the rotor travels four revolutions, with a root mean square error of six pockets. And suppose still further that once the ball is on the rotor, it travels 13 pockets with a root mean square error of eight pockets. Given these suppositions, you can predict that the ball will travel eight revolutions plus four revolutions plus 13 pockets from the launch point, or 13 pockets beyond that point. The root mean square error is the square root of five squared plus six squared plus eight squared. This turns out to be 11.2 pockets, well within the required error of less than 17 pockets. In this case, the prediction system would work.

However, I think you will find that when you collect this data, the errors at each stage are several times as large as I have used in this example. My own observation is that the dealer error in the number of revolutions for the ball spin is about 20 pockets for the more consistent dealers; it is much larger with a less consistent one. I also noticed that the rotor velocity is not nearly as constant as Kimmel and No Zero would like. That is because the dealer gives it an extra kick every few spins to rebuild its velocity.

It is also true that the deflecting vanes on the sides of the rotor add considerable randomness to the outcome, as do the frets or spacers between the pockets. The upshot is that I don't believe that any dealer is predictable enough to cause a root mean square error of less than 17 pockets. I'm willing to examine proof to the contrary, but I would be very surprised if anyone could ever produce it.

If a dealer dutifully practiced spinning the ball a fixed number of revolutions, and if a motor drive spun the rotor at a constant velocity, and if we have a very good way of deciding exactly which number is opposite the ball just as it is released, it might be barely possible to gain a small prediction advantage. I consider even that very unlikely. Dr. "No Zero" might be more aptly called Dr. "Knows Zero."

P.S. The standard way to test data like Kimmel's is with what statisticians call the Chi Square test. Epstein discusses this test for biased dice and roulette outcomes in *The Theory of Gambling and Statistical Logic*. Letting $n(k)$ be the number of times Kimmel observed the spacing k between successive outcomes, a count from his graph gives $n(0) + n(1) + \ldots + n(37) = 199$. (He apparently checked 200 outcomes to get 199 spacings between results.) The average result is $\bar{n} = 199/38 = 5.2368$. He indicates it is 5. The Chi square statistic, assuming all spacings are equally likely, is 58.98. There are 37 "degrees of freedom," and my calculator shows a 1.23 percent probability of a result at least as skewed as Kimmel's.

This percentage is unusual, but it is certainly not remarkable. About one time in 80, we would expect a pattern that is more skewed than the one Kimmel shows in his article. This example is probably his best; it is certainly not predictable enough upon which to base a system or theory.

A similar experiment would involve tossing a coin many times and noting what outcome occurs on the flip following a consecutive sequence of three heads. Suppose that after three heads there was a tail, and that this pattern—three heads, one tail—repeated seven times. The chance of that happening is one in 128, or less than one percent. It is a rarer occurrence than the one Kimmel shows us. But would you then base a theory of coin tossing on this apparent pattern and bet next time that a tail would occur after three heads? You would not if you understood the principles of probability and gambling.

Kimmel also says in his article that a minimum of 50 spins is needed to produce a reliable signature. We have seen an example of what he calls a strong dealer signature—based on 200 spins—and found that it does not pass the Chi square test. A sample of 50 spins is too small upon which to base any statistical conclusions. If you check the careful work of Wilson in the *Casino Gamblers Guide*, you will see that thousands of spins, sometimes even tens of thousands, were needed to draw accurate statistical conclusions.

In closing, I'll give you the perfect casino countermeasure to the strategy of the dealer's signature, pretending for the moment that the strategy worked. First, the casino halts the betting before the dealer spins the ball. Second, the dealer closes his eyes or looks away from the wheel when he releases the ball so that he has no knowledge of which number on the rotor is closest to the ball when it is launched. Then, for the reasons explained above, the result will be perfectly random.

Q: *In a letter to* Datamation *(Dec. 1979), Allan N. Wilson mentions papers you have published in connection with maximizing the rate of winning in betting. I would be grateful if you could send me references to these articles.*

R.T.

A: Refer to my December 1979 column in *Gambling Times*. The two basic mathematical articles that I have written on this subject are "Optimal Gambling Systems for Favorable Games," *Review of International Institute of Statistics* 37:3 (1969), 273-293 and "Portfolio Choice and the Kelly Criterion," *Proceedings of the 1971 Business and Economics Section of the American Statistical Association*, 1972, 215-224, reprinted in *Stochastic Optimization Models in Finance*, Academic Press, edited by W.T. Ziemba, S.L. Brumelle, and R.G. Vickson, pp 599-619. I also discuss applications to gambling in the original 1962 edition of *Beat the Dealer* and, with W.E. Walden, in "A Favorable Side Bet in Nevada Baccarat," *Journal of the American Statistical Association 61*, Part I (1969), 313-328.

You can find these articles by going to a large university or public library. If they don't have them in their collection, they can generally obtain them for you.

BJ PARANOIA

Continued from page 13

that shifting the index for betting increases from zero to five results in a decrease in average winnings of roughly two chips per hour. The standard deviation also decreases in this scenario, but the present advantage increases to 1.91 percent. These examples should convince the reader that the percent advantage index is far from being the most accurate predictor of casino winnings. Rows 7 and 8 demonstrate how average hourly winnings can drop by 40 percent while the percent advantage index increases from 1.29 to 1.91 percent.

Returning to row 1 of the table, columns 5 and 6 show the probability that a player will have won or lost chips after an hour of play. These entries show that in 55 out of 100 hours, a player using the Thorp system will leave the table a winner; in 45 out of 100 hours, he will leave a loser. These probabilities are very accurate estimates, and they are easily derived from

Continued on next page