The Mathematics of Gambling

Physical Prediction of Roulette IV

by Edward O. Thorp

The ball timing errors cause errors in predicting both the time and place the ball leaves the track. Even if the spiral path of the ball down the stator into the rotor is always the same in time and distance, this still yields errors in predicting when and where on the rotor the ball enters.

In our example, the equation for $t_d(T)$ is $t_d(T)=(20/3)\log_e((3/10)/(\exp(3T/20) - 1))=(20/3)\log_e(3x_d(T)/10 + 1)$. The error is approximately $\triangle t_d(T)=-(\triangle T)\exp(3T/20)/(\exp(3T/20) - 1)$. Thus again, if $T=0.8$ sec. and $\triangle T=0.012$ sec. $\triangle t_d(T)=-0.106$ sec. With a rotor speed of 0.33 r.p.s., this causes a rotor prediction error of 0.036 rev. or 1.3 pockets. In our example then, we measured T too large by 0.012 sec. This led us to believe the ball would leave the track at a point about 4.2 pockets before where it did. Therefore, we forecast impact on the rotor 4.2 pockets early. It also led us to believe the ball would leave the track sooner in time. Thus, we thought the rotor wouldn't revolve as far as it did. This made us forecast impact another 1.3 pockets early, for a total error of 5.5 pockets early. There are other important sources of error, so our final predictions were not this good. But they were good enough.

In summary, note that an error where $\triangle T$ is positive, i.e., we think T is bigger than it really is because we hit the switch early the first time or late the second time, leads us to think the ball is slower than it is. That makes us think $x_d(T)$ is shorter. Thus, we expect the ball at the rotor too soon and forecast impact on the rotor ahead of where it tends to occur. Conversely, if $\triangle T$ is negative (last on the first switch or early on the second), we think T is smaller, the ball is faster, and mistakenly forecast $x_0(T)$ and $t_d(T)$ as too big. Then we predict impact behind where it tends to occur.

The rotor angular velocity followed a law close to $r(t)=A\exp(-bt)$. A typical value for A was 0.33 rev./sec. The "decay" or "slowing down" constant b was very small. The rotor is massive and spins on a well-oiled bearing (on our casino wheel, it was the pointed end of a sturdy steel shaft). In the course of a minute or two, the slowing was hardly perceptible. (Note: Stroboscopic "beat frequency" techniques, plus an accurate clock, can quickly and easily give a very precise measurement of b and the slowing down.)

Let's take $b=-\log_e(10/11)/120$ or 0.000794/sec., which corresponds to a slowing down from 0.33 rev./sec. to 0.30 rev./sec. in two minutes. This seems like the right order of magnitude. To put the rotor position into the tiny computer we were going to build, we planned to hit a rotor timing switch once when the zero passed a reference mark on the wheel, and then hit the switch again when the zero passed the reference mark a second time. Since the rotor velocity was small and nearly constant, this was a less "sensitive" measurement. Therefore, we planned to do it first, shortly before the ball was spun.

How much error in the ball's final position (pocket) comes from rotor timing errors? Assume for simplicity that the rotor makes one revolution in about three seconds (.33 rev./sec.) and that we can neglect the slowing down of the rotor. Then, as in the ball timing, we might expect a typical (root mean square) size of about 11.2/1,000 seconds for the combined effect of the two errors. If the rotor really makes one revolution in 3.000 seconds, and we think it takes 3.0112 seconds, then in 30 seconds we think the wheel will travel 9.9628 revolutions whereas it really travels 10.0000 revolutions. Thus, the rotor goes .0372 rev. or 1.4 pockets farther than expected. Similarly, if we think the rotor takes 2.9888 seconds for one revolution, then in 30 seconds the rotor goes .0375 rev. or 1.4 pockets less than we expected.

Error Analysis

We now have a long list of sources for errors in the prediction of the ball's final position. They are:

E1 Rotor timing—use 1.4 pockets to illustrate.

E2 Ball timing—use 5.5 pockets to illustrate.

E3 Variations in ball "paths" on rotor (see Fig. 1, May issue). Error size is unknown, call it X.

E4 Ball path down stator: error due primarily to "deflectors" and varies with the type and placement. Use seven pockets to illustrate.

E5 Variation in distance ball travels on rotor: error due primarily to frets between pockets "spattering" ball, plus occasional very long paths along the rim of the rotor "outside" the pockets. Use six pockets to illustrate.

E6 Tilted wheel. (We didn't know about this yet.)

For illustrative purposes, assume the errors approximately obey the normal probability distribution. Then the standard deviation (typical size) of the sum of several errors in the square root of the sum of all the squared errors. For instance, using "pockets" as our unit, combined errors $E4 + E5$ have typical size $\sqrt{6^2 + 7^2}=\sqrt{85}=9.2$ pockets. Now add on the timing errors: $E_1 + E_2 + E_4 + E_5$ have typical size $\sqrt{1.4^2 + 5.5^2 + 6^2 + 7^2}=\sqrt{117.21}=10.8$ pockets. Thus the timing errors in this example cause very little additional error: just *10.8 − 9.2*, or 1.6 pockets.

Of course, we haven't added in *E3* yet and, if X is big enough, it could ruin everything. Possible variations in the ball orbit behavior on the stator were difficult for us to measure because we found it hard to tell at exactly what point the ball lost contact with the outer wall of the wheel. We also learned from both our own lab experiences and from watching in the casinos why the orbit varied somewhat. Once a drunken, cigar-smoking bettor knocked his ash onto the track. This was hard to clear out. It got on the ball and spread out on the track. That immediately changed the ball's behavior. Skin oil from our fingers or the croupier's would slowly "poison" ball and track and seem to affect the orbit behavior.

Typical Error E (No. of Pockets)	Percent Advantage Betting on Best	
	Pocket	Octant
0	3500.00	620.00
1	1278.53	611.06
2	610.69	467.86
3	376.52	328.65
4	258.12	236.98
5	186.76	175.71
6	139.09	132.62
7	105.00	100.89
8	79.41	76.65
9	59.54	57.60
10	43.77	42.38
11	31.19	30.18
12	21.24	20.52
13	13.54	13.03
14	7.73	7.37
15	3.47	3.24
16	0.46	0.30
17	− 1.62	− 1.72
18	− 3.01	− 3.07
19	− 3.90	− 3.94
20	− 4.46	− 4.49
21	− 4.81	− 4.82
22	− 5.01	− 5.02
23	− 5.13	− 5.13
24	− 5.19	− 5.19
25	− 5.23	− 5.23
26	− 5.24	− 5.25
27	− 5.25	− 5.25
28	− 5.26	− 5.26
29	− 5.26	− 5.26
30	− 5.26	− 5.26
∞	− 5.26	− 5.26

If we or the croupier gave the ball lots of axial "spin" (in the sense of tennis or ping pong), it could take several revolutions around the track before this abnormal spin energy was converted to orbit energy. (We named this effect after the famous quantum mechanics concept of "spin-orbit coupling.") On the other hand, the ball might be launched with no spin or backspin, so it would skid for a while before spin and orbit got "into synch."

Advantage Versus Error

Obviously, the greater the error, the less the advantage. If we assume the total prediction error *E* is (approximately) normally distributed, then we can construct a table showing the player's expected gain or loss as a function of *E*.

The table gives the results for a bet on the best pocket and also for a bet on the best "octant." The best octant is a set of five pockets, two on each side of the best pocket.

The table shows that, when the prediction error is normally distributed, the typical forecast error (standard deviation) must be 16 pockets or less, in order for the bettor to have an advantage. This is 16/38, or about 0.42 revolutions. This is true both for bets on the best pocket and the best octant. Since the best octant includes four pockets that aren't quite as good as the best, the advantage is somewhat less for a given typical error *E*. However, as we will see later in discussing the Kelly-Breiman system for money management, it is generally better for a small to medium-sized bankroll to bet the best octant.

Next month's column will answer some of the flood of questions you have been sending me. Roulette will be continued.