August '79 61

The Mathematics of Gambling

Physical Prediction of Roulette III

by Edward O. Thorp

I will now take you through a simplified version of what we first tried to do. Later, with that overview to guide us, I'll explain some of the modifications we had to make and describe our casino experiences.

First, let's consider part 1, the motion of the ball on the track. The actual function $x(t)$, which describes the number of remaining revolutions x versus the remaining time t, is theoretically very complex.

For this first simple discussion, let's suppose $x(t)=a\ exp(bt) + c$, where a, b, and c are constants and exp is the exponential function. This is one of the simplest mathematical functions that has the right "shape." (Note: Mathematical readers may wish to redo this discussion using the quadratic $x(t)=at^2 + bt + c$ to see the difference.)

I recall that the ball velocity at the point where it fell from the track was about 0.5 revolutions per second (r.p.s.) and that ten revolutions earlier it was about 2 r.p.s. Using this and the choice $t=0$ when the ball leaves the track gives $a=10/3$, $b=3/20$, and $c=-10/3$. Thus, $x(t)=10(exp(3t/20) - 1)/3$ in r.p.s., and this gives an angular velocity v in r.p.s. of $v(t)= \frac{1}{2} exp(3t/20)$. Figure 1 shows a graph of x(t).

Our first problem, and the key one, was to predict when and where on the stator the ball would leave the track. This problem was key because once we knew this, everything else except rotor velocity was a "constant." And rotor velocity is easy to measure in advance and incorporate into the prediction, as we shall see. Our method was to measure the time of one ball revolution. If the time were short, the ball was "fast" and had a long way to go. If the time were "long," the ball was "slow" and would soon fall from the track.

We hit a microswitch as the ball passed a reference mark on the stator. This started the electronic clock. This was at time t_1 (to go) with x_1 revolutions to go. (There are many such "marks" available on all actual casino wheels.) When the ball passed the reference mark the second time we hit the switch again, stopping the electronic clock. That was at time t_0 (left to go before the ball left the track), with x_0 revolutions left. The clock measured $t_1 - t_2$, the time T for one revolution (so $x_1 - x_0=1$). A calculation shows, for our illustrative $x(t)$ function, that $x_0(T)= 1/(exp(3T/20) - 1) - 10/3$. Thus, from T we can predict the number of revolutions until the ball leaves the track. For instance, if $T=1$ sec., we predict the ball will leave the track in $x_0(1)=1/(exp(3/20) - 1) - 10/3=2.85$ revolutions after the switch is hit the second time. If instead $T=\frac{1}{2}$ sec., then we predict $x_0(\frac{1}{2})=9.51$ revolutions.

Movie Experiments

The function $x(t)$ which we are using in this illustration is not the actual one. The actual $x(t)$ can be determined by a "movie experiment" like the ones I described earlier which I did in 1959 on my half-size wheel. To do this experiment today, get a full-size roulette wheel, a large clock which reads accurately in hundredths of a second or better, and a video camera or movie camera. Then take a movie of the orbiting ball. The successive frames give values for t and $x(t)$, which can be plotted to get an $x(t)$ curve like that of figure 1. Several movies should be made to see how much the $x(t)$ curve varies from one spin to another. This uncertainty is a source of error, much like the timing errors in determining T, that I'll discuss later on. These $x(t)$ errors can be incorporated into the theory in the same way as the timing errors. They each cause some uncertainty in the predicted $x_0(T)$ value. The data from the movie experiment can be improved if the camera frames are sychronized to a strobe so that the motion of both ball and clock is "stopped" rather than blurry. I didn't do this in my original movies, so I got a short blurry arc, instead of a ball, in each frame.

If an appropriate clock is not available, you can use a high quality phonograph turntable instead. These rotate at very uniform speeds which can be verified for your turntable with a strobe. Now get a stiff paper disc and mark the edges in equal small units. Number these units (much as you would a "circular" ruler) for ease in reading. Now place a thin fixed pointer just above the disc. When the disc rotates, you have a very accurate clock whose hand is fixed and whose face moves. If you use a paper disc of polar coordinate graph paper (glued, perhaps, to an old record), there will be 360 equally spaced degree marks.

At 33⅓ r.p.m., each mark is 1/200 sec. At 45 r.p.m., each mark is 1/270 sec., and at 78 r.p.m., each mark is 1/468 sec. On a 12-inch disc, the 360 marks will be spaced about a tenth of an inch apart so additional marks can be used or the pictures can simply be read to a fraction of an interval. Record test discs with equally spaced "spokes," for use with a strobe for testing turntables, are also available and can be used.

Timing Errors

Shannon and I used the switch which measured T to flash a strobe as well as start and stop the clock. We discovered the lights and the strobe flash "stopped" the ball at each of the two instants the switch was hit. This allowed us to see how much the ball was off the reference mark. Since we knew approximately how fast the ball was moving, we could tell about how much in time we were early or late in hitting the switch. This enabled us to correct the times recorded on the clock, thereby making the data much more accurate. We also learned from the visual feedback how to become much more accurate at timing.

Here's an illustration. Suppose the track of the wheel was 25 inches in diameter. (I don't have any of this equipment now so I'm remembering back almost 20 years and recalling about what the sizes, velocities, etc. seemed to be. They'll be close enough to be representative and good enough to show you how to do it all again, better for yourself if you want to.) Suppose the ball is ¾ inch in diameter and T, the time for one revolution, is 0.8 seconds. Then the track is 78.54 inches in length, or 98.17 ball diameters. If the ball center is one diameter away from the reference mark when the strobe flashes, then the timing error is about 1/98.17 of T or about 8/1000 of a second. There will be one of these errors when the

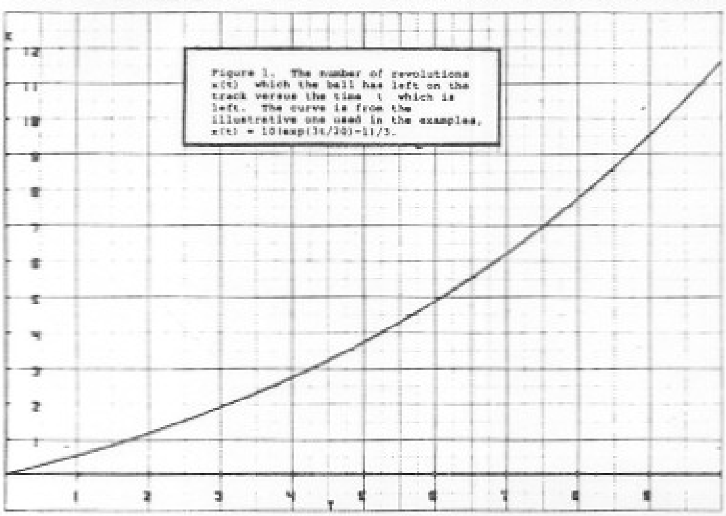

Figure 1. The number of revolutions x(t) which the ball has left on the track versus the time t which is left. The curve is from the illustrative one used in the examples, x(t) = 10(exp(3t/30)-1)/3.

switch is first hit and another when it is hit the second time. With practice we were able to reduce each error to a typical (root mean square) size of one ball diameter or about 8/1000 seconds. According to the theory of errors, the two errors together give a typical (root mean square) size of $8\sqrt{2}/1000$ or about 11.2/1000 seconds.

These errors would be unobservable in casino play, so we couldn't correct for them there. How much do they affect the prediction? (Math readers: $dx_0(T)/dT=-(3x_0(T)+10)^2/60$.) It can be shown that for the $x(t)$ of this example, the error $\Delta x_0(T)$ in the prediction of $x_0(T)$ due to an error ΔT in measuring T, is given by $\Delta x_0(T)=-(3x_0(T)+10)^2 T/60=-3\ T/\{20[\exp(3T/20)-1]^2\}$. For instance, if $T=0.8$ sec. and $\Delta T=0.012$ sec., we have a prediction error of $\Delta x_0(0.8)=0.11$ revs or 4.2 numbers on the wheel. In our illustration $T=0.8$ sec. means $x_0(T)=4.51$ revolutions to go. The time to go is $(20/3)\log_e(3x_0(t)/10+1)$ or 5.70 sec. We have somewhat less time than this to bet.

A Simple Casino Countermeasure

It should be clear that for this method to work, we have to time the ball (and rotor) before placing our potentially winning bets. (Earlier bets are losing, on average, so are only camouflage.) Thus, the casino must allow us to continue to bet for a time after the ball is launched. I have observed roulette wheels all over the world: Monte Carlo (our final goal), Nevada, Puerto Rico, Nice, Venice, and London. The practice has been, generally but not always, to allow bets until the ball was almost ready to fall off the track. This was much longer than we needed. Be warned again, though; all the casino needs to do to prevent our method is to forbid bets once the ball is launched. That simple perfect countermeasure is the Achilles heel of the system and a major reason why I never made a total effort to implement it. (People who use the system in casino play say the casinos don't catch on and don't use the countermeasure. But if the player is not really careful I would expect the casino to catch on.) GT

(To be continued.)

ASK OUR EXPERTS

Continued from page 8

the draw, and no one else called or saw the raise, he would be required to show that he has the required minimum holding or else he forfeits the pot. However, any other player can have whatever hand he chooses to have without any minimum holding, and he would not be required to show that he had jacks or better to claim the pot.

Q: *Are place bets working at a craps table on the come-out roll? I keep reading different and conflicting opinions about this, and I'd like you to set me straight.*

T.Y.W.
Miami, FL

A: In practically all casinos, place bets are not working and are "off" on the first, initial, or come-out roll. Once a point is established, then they work and are either lost or paid off, whichever the case may be.

Q: *In keno, does one particular number come up more frequently than any other number? If so, could you let me know what that number is?*

A.R.
Cleveland, OH

A: Assuming that the numbers in keno are selected at random, no one number in the long run will show more than any other number, but even if this wasn't the case, there is no way I could answer your question. Which keno game would you be talking about? In what casino?

Some astute players keep track of the numbers that come up in certain keno games, because, through the machinery or through the selection process used by the operator, the ping pong balls holding the numbers may show a bias, either through use, dents, or for some other reason. But, overall, the game of keno, per se, has no single number showing up more than any other number.

Q: *I recently read a book in which the author states that by 1979, the I.R.S. is supposed to report to Congress about the advisability of withholding taxes on gambling winnings. How can that be done at*