The Mathematics of Gambling:

Letters from Readers

by Edward O. Thorp

© 1979 Edward O. Thorp

Q: *I am currently compiling material for a special topics course on the mathematical and computational aspects of gambling, especially blackjack. But I have not yet been able to locate certain essential material. Specifically, could you provide me with references to where the problem of derivations from basic strategy as a function of point-count is considered in detail? My need is for information as to how the particular number matrix is derived for whatever count system is being utilized. For instance, say one is using the simple-point count A, 2, 8=0, 3 − 7=+1, 9, 10, J, Q, K=−1. How does one mathematically (or computationally) determine that one should stand on hard 16 if the true count is greater than or equal to 0, otherwise hit? Any documentary evidence you can direct me to on computations of this sort will be appreciated.*

J.C.

A: As far as I know, no one has published a detailed description of how the count system matrix (strategy table) is derived. There are two basic methods in use. The first, which I used for the 10-count system in *Beat the Dealer*, went as follows: I varied the number of 10s in the deck and held the remaining cards constant. I then plotted how the expectation for a given decision changed versus the 10-count ratio. I got a curve which generally was upward or downward sloping and crossed zero at just one point. That point was the decision point that I used in the strategy tables. That point would depend upon whether I used one full deck with just the number of 10s varying, two full decks with just the number of 10s varying, and so on.

The second method which has been used, and which I have used to some extent, is the simulation method. In the simulation method, you select random subsets of the deck and proceed in the same way. The "random" subsets might be selected by dealing hands in the same way as the casino and stopping periodically or by other methods. With the simulation approach, you don't get a smooth graph but rather a scatter of points which needs to be fit by a regression technique.

The numbers can be tested by another kind of simulation. Suppose, for instance, that plus 3 is the decision point. Now let the computer play a large number of hands in which the particular decision situation arises and in which plus 3 is the state of the deck. Then see whether the two decisions seem to yield approximately equal gains or losses. If instead there is a statistically significant difference, then plus 3 is in doubt and needs to be reexamined.

Q: *I write this letter with the reluctance of one who must admit failure. I firmly believe that "the house" can be beaten at blackjack, and I am convinced that the "10-count strategy" you created is one of the best—the best for me to achieve this goal and yet . . .*

Over the last two years, I have dealt myself, at home, approximately 10,000 hands of blackjack from one and two complete decks under today's conditions in Las Vegas casinos. During this period, I learned to keep a proficient running count of either one or two decks and to count 5s and aces in one deck and to "lean" in two decks, making very few errors. Throughout the 10,000 hands, I used the consecutive progressions 1-2-3 and 1-2-4 units, and, for the sake of expediency, I simulated head-on play.

The results of this experiment showed a loss of 500 units. I still cannot believe this.

When I was half through this experiment and showing an overall loss of approximately two percent

of the action, I decided to try out my abilities in actual play and went to Las Vegas with a fund of $300—150 units at $2 each. I played approximately 20 hours in three days. At the end of the three days, I had lost $160 (80 units).

The trip was not a complete loss because—mind you that I am not a gambler—I learned that:

a. The great majority of dealers are robots and are programmed only to deal cards.
b. Ninety percent of blackjack players throw their money away.
c. Casino management does its utmost to keep the players from concentrating.
d. It makes a big difference to play at a table with four, five or six other players—mostly bad—as opposed to playing head-on.
e. Compared to the speed I dealt at home, in actual casino play I can count and calculate very easily. I admit losing the count several times—but nothing to effect the outcome of my plays drastically.

What am I doing wrong? I cannot find any flaws in what I am doing, and I refuse to believe that a run of "bad luck" can last this long or that "personal bad luck" defeats what is mathematical fact. Of all my large bets (three or four units each), I lose, on the average, five out of ten; I win four with one push. Any comments on this?

R.L.

A: Yours is one of the very few letters that I have received from people who have had long-term negative results attempting to play the 10-count strategy. Here are several suggestions which might uncover the problem. First, you should be aware that even if you make few counting errors, if you convert one large bet per hour from a win to a loss by errors, this is enough to cancel much or even all of your advantage. That is because a good player has an edge of roughly one or two big bets per hour under typical procedures.

When practicing at home, I suggest that you not deal all the way through the deck but instead stop dealing when there are 5 to 15 cards left. Then confirm the accuracy of your card count by first writing down the count as you know it. Then spread the unused cards and see what the count really is. If your count is correct, good. If it is not correct, then the results of the hands that you played this time through the deck are not a test of the system and should not be included in your record of system results. I have found this method of confirming my card count very useful. A second question that your letter suggests to me is did you keep an accurate diary of your results? Not many people are likely to do this over a two-year period. Another question that arises in this letter is if you simply were "unlucky." The 10-count and other systems which give the player the advantage don't guarantee that he will win; they just make it more or less likely depending on the duration of play and other factors. I made some calculations which show that if you made enough counting errors to eliminate your advantage, there would be a two percent chance that you would get a result as bad as losing 500 units in 10,000 hands. That is "unlucky," but if 1,000 readers did this, 50 would have your experience or worse. We could check the "unlucky" hypothesis by keeping a tally of the number of one-unit bets, two-unit bets, and three-unit bets that are made and the net gain or loss.

Whatever is happening appears to have happened both in your home play and in the casino. At least the loss rates are similar. I might remind you that Beat the Dealer says not to begin serious play in the casino until you have won consistently at home. That is to be sure that everything is being done right in practice before you risk your money.

Stanley Roberts has a blackjack school in Hollywood, Ca., that you might wish to visit, and I believe they have computer programs that you could use to monitor your play and as a teaching device.

Q: I am a teacher of language arts on vacation in Las Vegas and not having a very good time of it. I have learned the complete point count, am able to subtract cards, ascertain a count, use the hi-lo in-

Continued on next page

dex, account for As and 5s, and size my bets. I am sufficiently keen to have been barred from the Barbary Coast my second day in town.

On page 65 of **Beat the Dealer**, there is a quiet and disarming sentence. "Since the favorable situations arise with greatest frequency at the end of the deck, shuffling up can sharply reduce the rate of profit."

Based on my hours of recorded practice hands, an empirical deduction tells me that for a counter to realize full benefit, 75 percent of the cards must be dealt; a shuffle point of 13 in a single, 26 in a double, and 52 in four decks. Is my deduction accurate? What rate of profit is to be expected if, say, a single deck were shuffled 20 cards from the end? Also, have you developed an advanced strategy similar to Revere's? If so, are the relevant particulars available?

The Frontier now uses six decks and deals only three, a prohibitive situation, I'm sure. However, I have the compensatory satisfaction of having learned something that once I believed impossible; an experience I can only liken to having flown with the Wright Brothers. If, indeed, I am still able to win, and card counting is not another charming oddity to be consigned to a sunny place in my personal aerospace museum, would you be kind enough to send me whatever may be beneficial?

P.H.

A: The rate of profit for card counters in blackjack depends jointly on three variables: (1) The overall favorability of the casino rules. A good indication of this is the house or player edge with basic strategy. See the discussion of rules variations on page 131 of *Beat the Dealer* (1966 edition). (2) The player's level of skill. Assuming that you do not make counting errors, you are at the expert level. (3) The number of decks of cards in the pack and the reshuffle point.

Typically, if a pack of cards is reshuffled half way through, the number of favorable situations is so few that it isn't worth the counter's time and trouble. This simple effective casino counter-measure could pretty well end the damage from counters.

Occasionally, the rules are favor-

able enough so that basic strategy gives you an edge off the top of the pack. Then you can win no matter what the shuffle point is, and card counting can increase your edge further.

I don't market blackjack strategies. See the *Gambling Times* Systems Shop for available strategies. The hi-low with aces, simplified if you wish by estimating remaining cards (or half decks), is almost as good and is, with Hi Opt I, the easiest to use and the cheapest ($2.45). The Hi Opt I is essentially a more complete, improved and exact version of the Hi-Low. It comes with instructions for using the ace side count, has two-deck and surrender strategies, and incorporates the strategy improvement of counting aces as 0 for strategy (but using them for bet sizing). For practical use in casinos, I think that the simple 0, +1, −1 count is much easier to use than systems with other point values.

If 75 percent of the cards in the pack are dealt, I think counting for profit is worthwhile. I agree that the Frontier situation you describe doesn't seem worth playing. **9**

Answers to Last Month's Gambler's Crossword

ACROSS
1. PARLAYS
4. MOSES
7. CROSS
9. INSIDER
10. RACING FORM
14. MARTIN
15. SCOOPS
17. AGGIES
18. ESP
21. HOT
22. IVY
23. SHOE
25. DOYLE
26. RELATES

DOWN
1. PACER
2. LAS
3. STIFFS
5. SIDE
6. SPREADS
8. SAN DIEGO
11. CARD
12. RACES
13. LOVE
14. MATCHED
16. SILVER
19. PRESS
20. STAY
24. HIT