The Mathematics of Gambling:

Cheating at Blackjack

by Edward O. Thorp

Various card counting systems give the blackjack player an advantage, provided that the cards are well shuffled and that the game is honest. But many methods may be used to cheat the player. I have been victimized by most of the more common techniques and have catalogued them in *Beat the Dealer*. The classic reference on cheating, *Expert at the Card Table*, may be obtained from the *Gambling Times* bookshelf.

One of the simplest and most effective ways for a dealer to cheat is to peek at the top card and then deal either that card or the one under it, called the second. A good peek can be invisible to the player. A good second deal, though visible to the player, can be done so quickly and smoothly that the eye generally will not detect it. Although the deal of the second card may sound different from the deal of the first one, the background noise of the casinos usually covers this completely. Peeking and second dealing leave no evidence. Because these methods are widespread, it is worth knowing how powerful they are.

Does even a top professional blackjack counter have a chance against a dealer who peeks and deals seconds? Consider first the simple case of one player versus a dealer with one deck. This is an extreme example, but it will illustrate the important ideas.

I shuffle the deck and hold it face up in order to deal practice hands. Because I can see the top card at all times, dealing from a face-up deck is equivalent to peeking on each and every card. I will deal either the first or second card, depending on which gives the dealer the greatest chance to win. I will

think out loud as an imaginary dealer might, and the principles I use will be listed as they occur. The results for a pass through one deck are listed in *figure 1*. There were nine hands, and the dealer won them all.

On hands one, two, four, six, eight and nine, the dealer wins by busting the player. Because there is only one player, it does not matter what cards the dealer draws after the player busts.

When there are two or more players, the dealer may choose a different strategy. If, for example, the dealer wishes to beat all the players but doesn't want to peek very often, an efficient approach is simply to peek when he can on each round of cards until he finds a good card for himself on top. He then retains this card by dealing seconds until he comes to his own hand, at which time he deals the top card to himself. That strategy would lead to the dealer having unusually good hands at the expense of the collective player hands; because some good hands have been shifted from the players, the player hands would be somewhat poorer than average.

A player could detect such cheating by tallying the number of good cards (such as aces and 10s) which are dealt to the dealer as his first two cards and comparing that total with the number of aces and 10s predicted by theory. In Peter Griffin's new book, *The Theory of Blackjack*, he describes how he became suspicious after losing against consistently good dealer hands. Griffin writes that he " . . . embarked on a lengthy observation of the frequency of dealer up cards in the casinos I had suffered most in. The result of my sample, that the dealers had 770 aces or 10s out

of 1,820 hands played, was a statistically significant indication of some sort of legerdemain." Griffin's tally is overwhelming evidence that something was peculiar. The odds against such an excess of ten-value cards and aces going to the dealer in a sample this size are about four in ten thousand (Z statistic 3.37).

Another approach the dealer might select is to beat one player at the table while giving everybody else normal cards. To do this, the dealer peeks frequently enough to give himself the option of dealing a first or second to the unfortunate player each time that player's turn to draw a card comes up. Dealing stiffs to a player so that he is likely to bust is, as we see from the chart in *figure 1*, so easy to do that the player has little chance.

If all dealers peeked and dealt seconds according to the cheating strategy indicated in the chart, I estimate that with one player versus the dealer, the dealer would generally win at least 95 percent of the time. With one dealer against several players, the dealer would win approximately 90 percent of the time. Anyone who is interested can get a good indication of what the actual numbers are by dealing a large number of hands and recording the results. If those results prove interesting, send them to me, and I will report them in this column.

The deadliest way a dealer can cheat is to win just a few extra hands an hour from the players. This approach is effective because it is not extreme enough to attract attention, or to be statistically significant and therefore detectable over a normal playing time of a few hours. For example, the odds in blackjack are fairly close to even for either the dealer or the player to win a typical hand. Suppose that by cheating the dealer shifts the advantage not to 100 percent but to just 50 percent in favor of the house. What effect does this have on the game?

If we assume that the player plays 100 hands, a typical total for an hour's playing time, and we also assume that the player bets an average of two units per hand, then being cheated once per 100 hands reduces the player's win by one

unit on the average. A professional player varying his bet from one to five units would probably win between five and 15 units per hour. The actual rate would depend upon casino rules, the player's level of skill, and the power and variety of winning methods that he employed. Let's take a typical professional playing under good conditions and assume that his win rate is ten units per hour and his average bet size is two units. Given those assumptions, being cheated ten times per hour or one-tenth of the time would cancel his advantage. Being cheated more than ten percent of the time would probably turn him into a loser.

Cheating in the real world is probably more effective than in the hypothetical example just cited, because the calculations for that example assume cheating is equally likely for small bets and big bets. In my experience, the bettor is much more likely to be cheated on large bets than on small ones. Therefore, the dealer who cheats with maximum efficiency will wait

Figure 1

Hand	Top Card	Card Dealt	Comment	Plr. Gets	Plr. Total	Dlr. Gets	Dlr. Total	Result
1	2	First		2	2			
	5	Second (4)	Dlr. trys for good card			4	4	
	5	Second (10)	Dlr. will have 9; Plr. gets stiff	10	12			
	5	First				5	9	
	4	First	Worsens Plr. stiff	4	16			
	5	Second (8)	Prevent Plr. 21	8	24 bust			Dlr. wins
	5	First	Doesn't matter			5	14	
	9	First	Doesn't matter			9	23 bust	
2	4	First		4	4			
	K	First				10	10	
	Q	First	Give Plr. stiff	10	14			
	J	Second (J)	J will bust Plr. (Second turns out to be J, too!)	J	24 bust			Dlr. wins
	J	First	Doesn't matter			J	20	
3	A	Second (3)		3	3			
	A	First				A	1, 11	
	3	First	Bldg. potential stiff	3	6			
	J	First				J	BJ	Dlr. wins
4	2	First		2	2			
	9	First	Would make Plr. 11, so Dlr. takes			9	9	
	K	First	Give Plr. stiff	K	12			
	Q	Second (3)	Guarantees Dlr. win			3	12	
	Q	First		Q	22 bust			
	8	First				8	20	Dlr. wins
5	5	First		5	5			
	J	First				J	10	
	A	Second (A)	Guarantees Dlr. win (Second was A, too!)	A	6,16			
	A	First				A	BJ	Dlr. wins
6	8	First		8	8			
	Q	First				Q	10	
	6	First	Give Plr. stiff	6	14			
	2	Second (3)				3	13	
	2	First	Give Plr. worse stiff	2	16			
	6	First		6	22 bust			Dlr. wins
	4	First				4	17	
7	7	First		7	7			
	6	Second (A)				A	1, 11	
	6	First		6	13			
	6	Second (K)				K	BJ	Dlr. wins
8	6	First		6	6			
	10	First				10	10	
	K	First	Give Plr. bad stiff	K	16			
	9	Second (2)	Dlr. must win			2	12	
	9	First		9	25 bust			Dlr. wins
	10	First	Doesn't matter	10		10	22 bust	
9	8	First		8	8			
	Q	First				Q	10	
	7	First		7	15			
	10	Second (5)	Dlr. must win			5	15	
	10	First		10	25 bust			Dlr. wins
	7	First				7	22	

They Shoot Up Horses

were done by a medical computer, the results would be known in minutes. In addition, legal medication in the proper dosages could be administered in the complex.

Earlier this year at a meeting of the Horsemen's Benevolent and Protective Association, Jockey's Guild, Thoroughbred Racing Association and National Association of State Racing Commissions, John Goodman, president of the NASRC, said, "It is my sincere desire that every member of the racing fraternity will cooperate fully to work out this vexing problem (medication) for the benefit of the industry we all care so much for."

The NASRC has formed a blue-ribbon committee designed to recommend a uniform medication rule. It is the fervent hope of all handicappers that racing's hierarchy, with the aid and cooperation of state legislatures, will devise and enforce a universally acceptable medication program, improve security systems, and implement better drug testing procedures. Measures such as these are sorely needed to restore credibility to the sport of kings. **9**

Mathematics of Gambling

until a player makes his top bet. Suppose that bet totals five units. If the cheat shifts the odds to 50 percent in favor of the house, the expected loss is 2-1/2 units, and just four cheating efforts per 100 hands will cancel a professional player's advantage. A cheating rate of five or ten hands per 100 will put this player at a severe disadvantage.

We can see from this that a comparatively small amount of cheating applied to the larger hands can have a significant impact on the game's outcome. This gives you an idea of what to look for when you are in the casinos and think that something may be amiss.

Q: *I've read the '66 edition of* **Beat the Dealer** *several times and found it very helpful. In table 4.1, p. 48, there is one puzzling statistic: player advantage of 1.62 percent when Q(10) is zero. I realize that this advantage accrues to a very special system, but virtually all counting methods with which I am familiar (both theoretical systems such as Griffin's "Optimal," p. 440,* **Gambling and Soc.,** *and practical systems such as Hi-Lo, Hi-Opt, and so on), value the "ten" cards as minus, which means that when Q(10) is zero, one is at a disadvantage. In your book, you note that this is a unique outcome, but could you discuss this result more fully?*

P.Z.

A: I discovered that when the composition of the deck changes by a relatively small amount, the change in player advantage is to good approximation a linear function of the change in the fraction or percentage of each card separately. For example, when one ace is used, the fraction of aces in a single deck changes from 4/52 to 3/51, causing a decrease in player expectation of 0.6 percent. Similarly, when other cards are used, there are changes in player expectation; to a good approximation, the effect of taking out several cards is a sum of the effects of taking the individual cards. That is what I mean by linear.

I also discovered that when the change in the composition of the deck is large, the linear approxima-

tion no longer holds very well in many cases. One of those cases occurs when all the 10s are gone, the special situation you mention in your letter. In fact, if you plot the player advantage versus the fraction of 10s in the deck, part of the graph will reflect a change that appears linear or proportional to the change in the fraction of 10s. But then the graph begins to curve. In the case of taking out 10s, it eventually curves around so much that the deck stops getting worse after the 10s fraction is sufficiently small and begins to gradually improve until—when all the 10s are gone—it is again good for the player. However, these calculations were made assuming that the player used the best strategy for all the 10s being out of the deck; if he used the basic strategy, his advantage would be less. Although I have not calculated what it would be, it is conceivable, though unlikely, that he might even have a disadvantage. **9**

Roberts' Rules

has proven himself, although he may unfairly attack you. This is because people who cannot subsist on their merits alone will generally collapse. I've seen it countless times. For example, our little ad has run for nine years in the *Los Angeles Times* Classified section. In that period, nearly 100 others have tried their hand at the game. They've gone the way of all flesh—none are still in business. When, however, that person reaches large audiences with his lies and deceit, a sensible businessman must protect his reputation.

Since dueling is no longer legal and thugs are definitely illegal, one is left with the big gun of litigation as the only viable alternative when faced with libelous statements. *Caveat libelor*—let the libelor beware.

Now that I have finally written this down, have given you my opinions on this matter, and know that you are reading it, I do feel a lot better. And as I examine the experience in retrospect, if I had to do it all over again, I would do the same thing. But I believe it would cost me a lot less and the other party a lot more. **9**