The Mathematics of Gambling:

Blackjack—Beware the Short Shoe

by Edward O. Thorp

Once again, I have recently heard complaints that cards have been missing from the pack in some casino blackjack games. This month, we'll discuss how you might spot this cheating method.

In 1962, I wrote on page 51 of *Beat the Dealer*, "Counting the . . . cards . . . is an invaluable asset in the detection of cheating because a common device is to remove one or more cards from the deck." Lance Humble discusses cheating methods for four-deck games dealt from a shoe in his International Blackjack Club newsletter. He says, "The house can take certain cards such as tens and aces out of the shoe. This is usually done after several rounds have been dealt and after the decks have been shuffled several times. It is done by palming the cards while they are being shuffled and by hiding them on the dealer's person. The dealer then disposes of the cards when he goes on his break." But cheating this way is not limited to the casino. Players have been known to remove "small" cards from the pack to tilt the edge their way. The casino can spot this simply by taking the pack and counting it; the player usually has to use statistical methods.

In the cheating trade, the method is known as the *short shoe*. Let's say the dealer is dealing from a shoe containing four decks of 52 cards each. In 52 cards, there should be 16 ten-value cards; the tens, jacks, queens and kings. Logically, in four decks of 208 cards, there should be 64 ten-value cards. I'll call all of these "tens" from now on. Casinos rarely remove the aces—even novice players sometimes count these.

Suppose the shift boss or pitboss takes out a total of ten tens; some of each kind, of course, not all kings or queens. The shoe is shortened from 64 tens to 54 tens, and the four decks from 208 cards to 198 cards.

The loss of these ten tens shifts the advantage from the player to the dealer or house. The ratio of others/tens changes from the normal 144/64=2.25 to 144/54=2.67, and this gains a little over one percent for the house. How can you discover the lack of tens without the dealer knowing it?

Here is one method that is used. If you're playing at the blackjack table, sit in the last chair on the dealer's right. Bet a small fixed amount throughout a whole pack of four decks. After the dealer puts the cut card back only, let's say, ten percent of the way into the four shuffled decks and returns the decks into the shoe, then ready yourself to count the cards. Play your hand mechanically, only pretending interest in your good or bad fortunes. What you're interested in finding out is the number of tens in the whole four-deck shoe.

Let's say the shift boss has removed ten tens. (Reports are that they seem to love removing exactly ten from a four-deck shoe.) When the white cut card shows at the face of the shoe, let's say that the running count of tens has reached 52. That means mathematically that if all 64 ten-value cards were in the shoe, then, of the remaining 15 cards behind the cut card, as many as 12 of them would be tens, which mathematically is very unlikely. This is how one detects the missing ten tens because the dealer never shows their faces but just places them face down on top of the stack of discarded cards to his right, which he then proceeds to shuffle face down in the usual manner preparatory to another four-deck shoe session.

Although at first the running count is not easy to keep in a real casino situation, a secondary difficulty is estimating the approximate number of cards left behind the cut card after all the shoe has been dealt. To practice this, take any deck of 52 cards and cut off what you think are ten, 15 or 20 cards, commit yourself to some definite number, and then count the cards to confirm the closeness of your estimate. After a while, you can look at a bunch of cards cut off and come quite close to their actual number.

In summary, count the number of tens seen from the beginning of a freshly shuffled and allegedly complete shoe. When the last card is seen and it is time to reshuffle the shoe, subtract the number of tens seen from the number that are supposed to be in the shoe—64 for a four-deck shoe—to get the number of unseen ten-value cards which should remain. If 54 ten-value cards were seen, there should be ten tens among the unused cards. Then estimate the number of unseen cards. You have to be sure to add to the estimated residual stack any cards which you did not see during the course of play, such as burned cards. Step four is to ask whether the number of unseen ten-value cards is remarkably large for the number of residual cards. If so, consider seriously the possibility that the shoe may be short. For instance, suppose there are 15 unseen cards, ten of which are supposed to be ten-values. A computation shows that the probability that the last 15 cards of a well-shuffled four-deck shoe will have at least ten ten-value cards is 0.003247 or about one chance in 308.

Thus the evidence against the casino on the basis of this one shoe alone is not overwhelming. But if we were to count down the same shoe several times and each time were to find the remaining cards suspiciously ten-rich, then the evidence would become very strong. Suppose that we counted down the shoe four times and that each time there were exactly 15 unseen cards. Suppose that the number of unseen tens, assuming a full four decks, was 9, 11, 10, and 13 respectively. Then referring to *table 1*, the probabilities to six decimal places are $H(9)=.014651$ to have nine or more unseen tens, and for at least 11, 10, and 13 respectively, the chances are $H(11)=.000539$, $H(10)=.003247$, and $H(13)=.000005$. These correspond to odds of about 1/68, 1/1,855, 1/308 and 1/200,000 respectively. The odds against all these events happening together is much greater still. In this example, the evidence strongly suggests that up to nine ten-value cards are missing. There can't be more than nine missing, of course, because we saw all but nine on one countdown.

If the casino shuffles after only 104 cards are seen, it is not so easy to tell if ten ten-value cards were removed. We see this as follows: Suppose (Hypothesis I) that the shoe really has four complete decks. Then the number X of unseen ten-value cards among the 104 cards (two decks) not seen will average 32. In the general case with U unseen cards, T tens in the whole pack, and N non-tens in the whole pack, the average value A of X is given by $A=UT/(N+T)$. In our example, $U=104$, $T=64$ and $N=144$, so we get $A=104 \times 64/208 =32$. But there will be a fluctuation around this number. Mathematicians use the standard deviation S to measure this fluctuation. The formula is $S^2=[UTN/(T+N)^2]\{1-(U-1)/(N+T-1)\}$.

For our example, $S^2=[104 \times 64 \times 144/208^2]\{1-103/207\}=11.1304$, so $S=\sqrt{11.1304}=3.3362$. To a good approximation, X is "normally distributed" with mean $A=32$ and standard deviation $S=3.3362$.

Now, suppose instead (Hypothesis II) that the deck has ten ten-value cards removed. Then $U=94$, $T=54$ and $N=134$. If Y is the number of unseen cards, we have the real A=25.6364, but we think there are ten more ten-value cards. So assuming incorrectly that no ten-values are gone, the number that we deduce for Y has an average of $A+10=35.6364$. The real S^2 for Y is $94 \times 54 \times 134/198^2\{1-93/197\}=\sqrt{9.1593}$, so $S=3.0264$.

What we want to know is whether to believe Hypothesis I ("null hypothesis") or Hypothesis II. This is a classic statistics problem. It turns out that in order for us to have a good chance to believe the correct hypothesis, the A value for X and Y need to be at least two and preferably several S units apart. In this example, they differ by only $35.6364-32=3.6364$, which is about one S unit. Of course, repeated countdowns of this same shoe will again increase our ability to tell whether the shoe is short.

This discussion should make it clear that the method suggested is generally not able to easily spot the removal of ten-value cards unless the shoe is counted several times or is dealt down close to the end.

One of the interesting ironies of the short shoe method of cheating players is that neither the shift boss nor the pitboss—the latter bringing the decks of cards to the dealer's table—need tell the dealer that his shoe is short. Thus, the dealer doesn't necessarily have to know that he's cheating. After all, he's just dealing. It's an open question how many dealers know that they're dealing from a short shoe.

Reports are that the short shoe is a frequent method that casinos use in cheating at blackjack using more than one deck. The tables with higher minimums (say $5) are more tempting candidates for short shoes than those with the lower minimums.

An experienced card counter can improve the method by counting both tens and non-tens. Then he'll know exactly how many unseen cards there are, as well as unseen tens. Table 1 can then be used with greater confidence.

In practice, you don't need to count through a shoe while betting (and thus losing money in the process) to find out that the casino is cheating. If you suspect foul play, count while standing behind the player to the dealer's right.

You might easily catch a short shoe by simply counting all the cards that are used, whether or not you see what they are. Then if the remaining cards, at the reshuffle, are few enough so you can accurately estimate their number, you can check the total count. For instance, you count 165 cards used and you estimate that 31 ± 3 cards remain. Then there were 196 ± 3 cards rather than the 208 expected, so the shoe is short.

A casino countermeasure is to put back a 4, 5 or 6 for each ace or ten-value card removed. Then the total number of cards remains 208, and the casino gets an even greater advantage than it would from a short shoe.

TABLE 1. FIFTEEN UNSEEN CARDS

K Number Of Unseen Ten-Value Cards	P(K) Probability Of Exactly K Unseen Tens	H(K) Probability At Least This Many Unseen Tens
0	.003171	1,000,000
1	.023413	.996829
2	.078818	.973416
3	.160423	.894598
4	.220732	.734176
5	.217437	.513443
6	.158380	.296006
7	.086431	.137626
8	.036132	.050782
9	.011404	.014651
10	.002707	.003247
11	.000475	.000539
12	.000059	.000065
13	.000005	.000005
14	.000000	.000000
15	.000000	.000000

Cards do get added to the deck, and there's a spooky coincidence to illustrate this. On page 51 of *Beat the Dealer*, I wrote in 1962, "One might wonder at this point whether casinos have also tried adding cards to the deck. I have only seen it done once. It is very risky. Imagine the shock and fury of a player who picks up his hand and sees that not only are both his cards 5s, but they are also both spades." And then 15 years later in 1977, a player in a one-deck game did get a hand with two of the same card—the 5 of spades. Walter Tyminski's casino gaming newsletter, *Rouge et Noir News*, reported on page 3 of the June 15, 1977 issue, "What would you do if the player at your right in a single blackjack game had two 5 of spades? Nicholas Zaika, a bail bondsman from Detroit, had that experience at the Sahara in Las Vegas on May 24 at a $5 minimum table.

"Zaika wasn't in the best of humor because he had earlier lost $594,000 at other Sahara tables, by far the largest loss he has ever experienced. Zaika had the blackjack supervisor check the cards and there were 53 cards in the deck, the duplicate being the 5 of spades . . . The gamer has engaged the services of Las Vegas attorney George Grazadei to pursue claims he feels that he has against the casino . . .

"The Sahara denies any wrongdoing and says that it is cooperating fully with the investigation . . . Players aren't likely to introduce an extra 5 because the presence of the extra 5 favors the house and not the player." The case remains in limbo as of this writing.

Suppose instead of just counting tens used and total cards used, you kept track of how many aces, 2s, 3s, queens, kings, and so on were used. This extra information should give the player a better chance of detecting the short shoe. The ultimate proof would be to count the number of each of the 52 types of cards which have been used. Mathematical readers might wish to investigate effective statistical or other ways of using information for detecting shoes in which the numbers of some of the cards have been changed.